Capital Asset Pricing Models with Default Risk: Theory and Application in Insurance

YUEYUN CHEN,* ISKANDAR S. HAMWI,** AND TIM HUDSON**

Abstract

The Capital Asset Pricing Model has been used frequently to derive a fair price of insurance. But the use of this model overestimates insurance premiums because it does not account for the insolvency risk of insurers. This paper examines how the insurance price should be fairly adjusted when insurers' default risk is considered. It develops a model which shows that fair insurance premiums are lower when insurance firms have a positive probability of being insolvent. Using data of property liability insurers during the period from 1943-99, the paper further estimates the effects of the insolvency risk on insurers' underwriting profit rate. It shows that the incorporation of the default risk of insurers in the model, by significantly reducing the required price for insurance, would lead to lower profit potentials. Some writers argue that including the insolvency risk when calculating insurance premiums is not so necessary because of the existence of states' guaranty insurance funds which protect consumers. However, as shown in the paper, these funds have provided inadequate protection to consumers. Therefore, because of the increase in the number of insolvencies in recent years, and because of the limited coverage provided by states' guaranty funds, it seems that considering the insolvency risk in insurance pricing has become very necessary. *(JEL G22); Int'l Advances in Econ. Res., 9(1): pp. 20-34, Feb 03.*

Introduction

Using the Capital Asset Pricing Model (CAPM) developed by Sharpe [1964], Litner [1965], and Mossin [1966], one is able to estimate the underwriting beta and further obtain the fair rate of return for insurance firms, as was previously done by Biger and Kahane [1978], Fairley [1979], Hill [1979], Bronars [1985], and Cummins and Harrington [1985]. The rate of return derived from such models is the competitive rate in the sense that all systematic risks an insurer faces are fairly compensated. The insurer is also compensated for its expected loss as well because of the relation between the rate of return of underwriting and the loss ratio.[1]

The rate of return and thus, the insurance premium derived using the CAPM, however, is overestimated because the model does not consider the insolvency risk of insurance firms.[2] Using such an overestimated rate of return to regulate the insurance industry will encourage investors to increase capital inflow to the industry. That in turn will cause over-supply of insurance which may lead to inefficiency.

In this paper, it is assumed that an insurance firm faces the risk of insolvency. As a result, policyholders' claims may not be fully recovered. With this assumption, the paper derives the formula for the fair price of underwriting using the CAPM. It shows that fair insurance premiums are lower when insurance firms have a positive probability of being insolvent. Using data of property liability insurers during the period from 1943-99, the paper further

*HSI LAI University and **University of Southern Mississippi—U.S.A.

estimates the effects of the insolvency risk on insurers' underwriting profit rate. It shows that the existence of the default risk of insurers significantly reduces the required underwriting rate of return. The paper also indicates that the traditional CAPM without including insolvency risk is misspecified and that the estimated underwriting beta is biased.

The Capital Asset Pricing and Option Pricing Models in Insurance

One main feature of this paper is combining the CAPM with the Option Pricing Model (OPM) for use in the insurance case. Such an approach alleviates the major weakness of solely using the CAPM while keeping the model simple and testable.

One distinction between the CAPM and OPM is that the CAPM ignores the insolvency risk of firms while the OPM explicitly considers such a risk. Since each firm has a positive probability of being insolvent, the results derived from the traditional CAPM have been questioned [Doherty and Garven, 1986; Brown and Hoyt, 1995]. An insurance firm earns premiums from consumers when it issues policies to them and invests some of its premiums in the financial market. Policyholders' claims, however, may not be fully paid if the insurer is declared to be insolvent because the insurer has a limited liability. Therefore, there is some value of insolvency to the insurer. The insurance price if calculated without including such a value will be unfair to consumers.

Researchers have long noticed the weakness of the CAPM due to its exclusion of insolvency risk. The problem was first pointed out by Fairley [1979]. Brown and Hoyt [1995] further urged that future research dealing with the CAPM should aim to incorporate the insolvency risk because of the significant relationship found between the insolvency rate and underwriting results. There are some reasons for excluding the insolvency risk in the CAPM as Fairley [1979] indicated. One is that the proper treatment of the insolvency risk in the context of the CAPM might be difficult. Fairley also thought that including the insolvency risk in the CAPM is not so urgent because of the existence of guaranty funds which he claims will provide adequate protection to consumers and also because the actual dollar volume of insolvency is small. However, the circumstances of the insurance market have changed greatly in the past two decades; in particular, the frequency of and the losses from insolvencies of insurance firms have increased dramatically since 1984.[3] From 1984-98, 480 property and liability insurers became insolvent[4] and, as a result, many policyholders' claims have not been recovered or fully recovered because of certain features of states' guaranty funds. A state's guaranty fund will not cover unpaid claims by insurers that are not licensed in that state. In addition, claim payments from a state's guaranty fund have upper limits. The total state's funds available to pay claims in a particular line of insurance (such as automobile insurance) are limited to the total assessment collected from all insurers doing business in that line and many states have annual assessment caps and usually do not start the levy of assessments to raise funds to pay claims until a failure has already occurred. The maximum assessment each insurer must pay in most states is two percent of total premiums written. In other words, if a state's total automobile insurance premiums written by all automobile insurers is $1 billion each year, then, the maximum guaranty fund assessment from all automobile insurers will be $20 million per year. If, in a given year, the total deficit of insolvent automobile insurers exceeds $20 million, some claims will not be paid in that year. Instead, they will be paid over the next few years depending on the availability of the state's fund. Furthermore, the payment from a state's guaranty fund to each policyholder is usually subject to a prescribed limit. In property liability insurance, most states set $25,000 as the maximum amount a policyholder can receive from the state fund.

Because of the increases in the number of insolvencies in recent years and because of the limited coverage provided by states' guaranty funds, it seems that considering the insolvency risk in insurance pricing has become more necessary. In particular, since the fair rate of insurance (premium rate) derived from the CAPM has been used in insurance regulation, (for instance, Massachusetts sets up its rate regulation using the CAPM [Derrig, 1986; Hill and Modigliani, 1986]), employing an appropriate method to include the insolvency risk in insurance rate-making is pertinent. Furthermore, with the inclusion of insolvency risk, the CAPM could also be useful in evaluating the adequacy of the various states' guaranty funds and the effectiveness of states' solvency regulations. The effects of such funds and the regulations on insurance rate-making can be analyzed by comparing two different fair rates; one with and one without the effect of having such funds and regulations. As an example, suppose that with solvency regulations, an insurer has a lower probability of insolvency and its deficit will, as a result, be smaller when it is insolvent. Then, one can derive a fair price of insurance which reflects the effect of such regulations. On the other hand, one can determine a fair price without regulations. The difference between these two rates is the value of solvency regulations to consumers. Consumers are charged more under solvency regulations because they are better protected.

Fair Pricing of Underwriting with a Default Risk

In this section, the fair rate of return which investors in an insurance firm could obtain is derived assuming that the insurer has a limited liability and faces an insolvency risk. The basic assumption implicitly imposed in the derivation is that the CAPM holds. In other words, it is assumed that investors in the insurance industry make decisions solely based on the expected return and variance of an investment.

The Model

Assume that an insurance firm has capital (surplus) K and writes policies with total premiums P. The total claim X, paid at the end of the period, is a random variable with a mean. The insurer invests all its capital (including its surplus and retained premiums) in a financial market with the rate of return r_i. Let V denote the insurer's net asset or value and its profit, then one has $V = (K + P)(1 + r_i) - X$. The insurer will be insolvent when its net asset is negative. Since the insurer has limited liability to pay its policyholders' claims, the actual profit of the insurer will be $\Pi = -K$ if it is insolvent and $\Pi = V - K$ if the insurer is solvent. In other words, one has:

$$\Pi = V - K + Max(0, -V) \tag{1}$$

or

$$\Pi = V - K + OV \quad , \tag{2}$$

where $OV = Max(0, -V)$ is the option value of insolvency of the firm. Equivalently, denote $\pi = \Pi/K$ as the profit rate. Then, one has:

$$\pi = \left(1 + \frac{P}{K}\right)(1 + r_i) - \frac{X}{K} - 1 + OV/K = \left(1 + \frac{P}{K}\right) r_i + \frac{P}{K} r_u + \frac{P}{K}\frac{OV}{P} \quad ,$$

where $r_u = 1 - \frac{X}{P}$ is the underwriting profit rate.

$$E(\pi) = \left(1 + \frac{P}{K}\right) E(r_i) + \frac{P}{K} E(r_u) + \frac{P}{K}(E\frac{OV}{P}) \quad , \tag{3}$$

however, from the CAPM, one has:

$$E(\pi) = r_f + \beta[E(r_m) - r_f] \quad , \tag{4}$$

and

$$E(r_i) = r_f + \beta_i[E(r_m) - r_f] \quad , \tag{5}$$

where $\beta = Cov(r_m,)/Var(r_m)$ and $\beta i = Cov(r_m, r_i)/Var(r_m)$. Furthermore, from the definition of the profit rate, one has:

$$\beta = (1 + \frac{P}{K})\beta_i + \frac{P}{K}\beta_u + \frac{P}{K}\beta ov/p \quad , \tag{6}$$

where β_u is the underwriting beta and $\beta ov/p$ equals $Cov(\frac{OV}{P}, r_m)/Var(r_m)$.

By substituting (6) into (4), and further substituting (4) and (5) into (3), one has:

$$\begin{aligned} & r_f + [(1 + \frac{P}{K})\beta_i + \frac{P}{K}\beta_u + \frac{P}{K}\beta ov/p][E(r_m) - r_f] \\ = \quad & (1 + \frac{P}{K})[r_f + \beta_i(E(r_m) - r_f)] + \frac{P}{K}E(r_u) + \frac{P}{K}(E\frac{OV}{P}) \quad . \end{aligned}$$

Rearranging the above equation, one gets:

$$E(r_u) = -r_f + \beta_u(E(r_m) - r_f) - [E\frac{OV}{P} - \beta ov/p(E(r_m) - r_f)] \tag{7}$$

or

$$E(r_u) = -r_f + \beta_u(E(r_m) - r_f) - Vp \quad , \tag{8}$$

where $Vp = E\frac{OV}{P} - \beta ov/p(E(r_m) - r_f)$ is the market value of option per unit premium. When the insurer has no default risk, $OV = 0$ and $\beta ov/p = 0$, thus one has:

$$E(r_u) = -r_f + \beta_u(E(r_m) - r_f) \quad . \tag{9}$$

This is exactly the same result obtained by D'arcy and Doherty [1988].

Notice that $r_u = 1 - \frac{X}{P}$, and $\beta_u = -Cov(X, r_m)/[P\ Var(r_m)]$.
From (7) one has:

$$P = (\bar{X} - \lambda Cov(X, r_m) - [E(OV) - \beta ov(E(r_m) - r_f)])/(1 + r_f) \tag{10}$$

or

$$P = (\bar{X} - \lambda Cov(X, r_m) - TVp)/(1 + r_f) \quad , \tag{11}$$

where $\lambda = (E(r_m) - r_f)/Var(r_m)$ is the market risk premium and $TVp = E(OV) - \beta ov(E(r_m) - r_f)$ is the total market value of option. Again, when the insurer has no default risk, $OV = \beta ov = 0$, so:

$$P = (\bar{X} - \lambda Cov(X, r_m))/(1 + r_f) \quad . \tag{12}$$

This is consistent with Bronars [1985], Hill [1979], Fairley [1979], and Biger and Kahane [1978].

A firm will have a low probability of insolvency when the economy is in a good condition, so $\beta ov/p \leq 0$; in addition, $OV \geq 0$. As a result, one has $Vp \geq 0$ and $TVp \geq 0$; so $E(r_u) \leq -r_f + \beta_u(E(r_m) - r_f)$ and $P \leq (\bar{X} - \lambda Cov(X, r_m))/(1 + r_f)$. $E(r_u) = -r_f + \beta u(E(r_m) - r_f)$ or $P = (\bar{X} - \lambda Cov(X, r_m))/(1 + r_f)$ only when $OV = 0$. In other words, the existence of the insolvency risk of insurance firms lowers the required underwriting profit rate and also the insurance premiums.

$\beta u[E(r_m) - r_f]$ is the risk premium of underwriting, so (9) implies that the expected profit rate of underwriting is equal to the underwriting risk premium minus the market value of the insolvency per unit premium and also minus the risk-free rate of return. r_f is subtracted because the insurer is borrowing money from its policyholders at the beginning of the period. In (11), $\bar{X}$ is the expected losses, $\lambda Cov(X, r_m)$ is the firm's risk premium for underwriting. Thus, (11) shows that the total insurance premiums equal the expected losses minus the underwriting risk premium and the total market value of insolvency, all discounted at the riskless rate of interest.

Extension of the Model

In the above derivation of the fair insurance premium, it is assumed that there is no underwriting cost and that all earned premiums collected at the beginning of the period can be invested. Now, suppose that the firm's total underwriting expenditures is C at the beginning of the period and that earned premiums are only invested by the proportion. Then, the firm's value will be:

$$V = (K + \delta(P - C))(1 + r_i) + (1 - \delta)(P - C) - X \quad .$$

Define $r_u = 1 - \frac{X}{P} - \frac{C}{P}$ as the underwriting profit rate. By repeating the same procedures as was done in the original derivation, one will have:

$$E(r_u) = -\delta(1 - c)r_f + \beta u[E(r_m) - r_f] - [E\frac{OV}{P} - \beta ov/p(E(r_m) - r_f)] \quad , \tag{13}$$

where $c = \frac{C}{P}$ is the average cost of underwriting per premium. Further, one has:

$$P = C + (\bar{X} - \lambda Cov(X, r_m) - [E(OV) - \beta ov(E(r_m) - r_f)])/(1 + \delta r_f) \quad . \tag{14}$$

Equations (13) and (14) show that the insurance firm needs higher compensation when a lower proportion of its premiums is being invested.

Furthermore, one may adjust the fair pricing to account for corporate tax liability. Assume that the statutory marginal tax rate is T. Let the value of $\theta_1 T$ represent the average tax rate applied to the firm's investment income where $0 \leq \theta_1 \leq 1$. Now let $\theta_2 T$ represent the effective tax rate on statutory underwriting profit. Then, one can redefine the expected return to equity holders associated with any given rate of underwriting profit $E(r_u)$, as:

$$E(\pi) = [1 + \delta\frac{P}{K}(1 - c)](1 - \theta_1 T)E(r_i) + \frac{P}{K}(1 - \theta_2 T)E(r_u) + \frac{P}{K}(E\frac{OV}{P})(1 - \theta_2 T).$$

Finally, one has:

$$\begin{aligned} E(r_u) &= -\delta(1 - c)r_f\frac{1 - \theta_1 T}{1 - \theta_2 T} + r_f\frac{\theta_1 T}{(1 - \theta_2 T)\frac{P}{K}} \\ &\quad + \beta_u\left(E\left(r_m\right) - r_f\right) - \left[E\frac{OV}{P} - \beta ov/p\left(E\left(r_m\right) - r_f\right)\right] \quad , \end{aligned} \tag{15}$$

and

$$P = C + (K\theta_1 r_f T)/[(1-\theta_2 T) + \delta r_f(1-\theta_1 T)] + (\bar{X} - \lambda Cov(X, r_m) - [E(OV) - \beta ov(E(r_m) - r_f)])/[1 + \delta r_f \frac{1-\theta_1 T}{1-\theta_2 T}] \quad . \tag{16}$$

Factors Affecting Insurers' Insolvency

An insurer's insolvency risk has a positive value to the insurer. Many factors affect an insurer's insolvency. Economic and market conditions such as the interest rate, unanticipated inflation, the number of insurers, the underwriting cycle and the whole industry's combined ratio (defined as the sum of the loss ratio and the underwriting expense ratio) all affect an insurer's insolvency [Brown and Hoyt, 1995]. The other factors affecting insolvency are firm-specific characteristics. They include policyholder's surplus ratio, the size of the firm, the organizational form of the firm, and the amount of reinsurance ceded. Munch and Smallwood [1980] find that capital and surplus requirements are the most effective means for reducing the frequency of insolvency. Mayers and Smith [1981] show that there are more severe agency problems in stock insurance firms than in mutual insurance firms; as a result, stock insurers will be more risk-taking than mutual insurers. Chen, et al. [2001] find that a less solvent insurer will use more reinsurance because of its difficulty in raising needed capital in the financial market and that excessive use of reinsurance tend to signal an insurer's risk of insolvency.

Bias in Estimating the Underwriting Beta

As pointed out in the introduction, many researchers have estimated the underwriting beta using the CAPM. Fairley [1979] finds the beta of 0.17 for the period 1971-75. Hill and Modigliani [1986] obtain an average beta between 0.16 or 0.27 for ten firms during 1970-76. Cummins and Harrington [1985] report an average beta of 0.12 for the period from 1970-75. All these estimated betas are very small. In fact, some researchers (for example, Hill [1979]) concluded that the estimated underwriting beta is not significantly different from zero.

As noted by Hill [1979], estimated betas are biased when accounting profits are not properly measured. The current study suggests that the bias can also result from not including a default risk in the model. Note that (8) has one more term than (9) which indicates the existence of misspecification problem in the model without the default risk. In other words, the estimated beta is biased when the model ignores any default risk.

Equation (8), shows that the estimated coefficient from regressing the underwriting profit rate, r_u, on the rate of return of the market portfolio, r_m, is the estimate for $(\beta_u + \beta ov/p)$ and not for β_u. Therefore, even if the estimated coefficient is close to zero as most previous studies obtained, the underwriting beta could be larger. Let the estimated coefficient from the regression be b_m, then the estimated underwriting beta is $(b_m - \beta ov/p)$; a value which is greater than b_m for $\beta ov/p \leq 0$.

Insurance Regulation and Insurance Rate Setting

Because of the financial difficulties experienced by the insurance industry in recent years, more federal intervention in insurance regulations such as those dealing with solvency issues has been called for to better protect consumers and the insurance industry [U.S. Congress, 1999]. Currently, insurance firms are subject to state regulation. To maintain solvency of insurance firms each state requires a minimum amount of capital and surplus. Also, many states require that premium rates be reasonable, adequate, and fair in order to avoid excessive competition or over pricing of the insurance product. Many studies have analyzed the effects of solvency regulation on the insurance market. On the one hand, some studies show that solvency regulations, by imposing the minimum capital and surplus requirements, are effective in reducing the number and costs of insolvencies [Munch and Smallwood, 1980; Lee, 1994]. On the other hand, other studies show that solvency regulations may lead to a higher price of insurance and hinder insurance market growth because it will discourage capital inflow into that market [Winter, 1992].

This study, however, indicates that rate regulation based on the traditional CAPM does not necessarily restrict capital inflow into the insurance industry. In contrast, it may encourage it because premium rates will be overestimated when the default risk is ignored. The minimum capital and surplus requirements may restrict capital inflow to the insurance industry. But, as long as invested capital in the industry is sufficient and fairly compensated, the adverse effect of these requirements on capital inflow will be minimized. In other words, provided insurance premiums are adjusted fairly according to the formula suggested in this study, solvency regulations do not necessarily prohibit capital inflow. Insurance premiums are also affected by market competition. Solvency regulations lead to fewer firms, which in turn cause insurance premiums to be higher because of the supply-side effect [Winter, 1992]. But, the probability of solvency of insurance firms will be increased when there are fewer firms [Brown and Hoyt, 1995] and that will further raise insurance premiums. As a result, more investors will be attracted to the insurance industry and that will ultimately cause the industry to be more competitive, thereby lowering insurance premiums. Thus, solvency regulations could affect capital inflow and raise insurance premiums, but when insurers are fairly compensated for their systematic risks, the outcome of such regulation on the insurance market could be different.

Underwriting Risk, the Value of Option, and Fair Pricing

As previously defined, $OV = Max(0, -V) = Max(0, X - (K + P)(1 + r_i))$, where X is the total loss, K is the total net capital (or surplus), P is the total premiums written and r_i is the rate of return from investing. The value of the option denoted by TVp will be a function of capital K, premium P and the distributions of both the loss X and the investment return r_i. It is easy to verify that the value of the option will be lower when the insurer's own capital is increased because such an increase will lead the firm to have a lower chance of becoming insolvent. Similarly, an increase in the investment return will lower the value of the option. But the effects of the total premiums on the option value are more complicated. Without considering its effect on total loss, the increase in total premiums will lower the option value. However, since the increase of a firm's total premiums usually implies that the firm issues more policies, its total loss may, as result, also increase. To avoid the problem associated with the existing relationship between total loss and total premiums, the value of option per unit premium denoted by $\frac{OV}{P}$ needs to be considered. In fact, in the fair pricing of underwriting (equations 8 and 9), it is the value which is relevant.

From the definition, $\frac{OV}{P} = Max(0, -V/P) = Max(0, x - (1 + k)(1 + r_i))$, where $x = \frac{X}{P}$ is the loss ratio and $k = \frac{K}{P}$ is the policyholder's surplus ratio. Both the loss ratio x and investment return r_i will be random variables and do affect the insurer's value of the option. An insurer's policyholder's surplus ratio could vary over time, but it is assumed here that such a ratio is not treated as random variable in pricing insurance because of the state's mandated minimum surplus ratio requirement.

It is obvious that the market value denoted by Vp of the option $\frac{OV}{P}$ will be lower when the policyholder's surplus ratio is raised. In addition, the distributions of both the loss ratio and investment return will affect the option value. However, unlike life and health insurers,

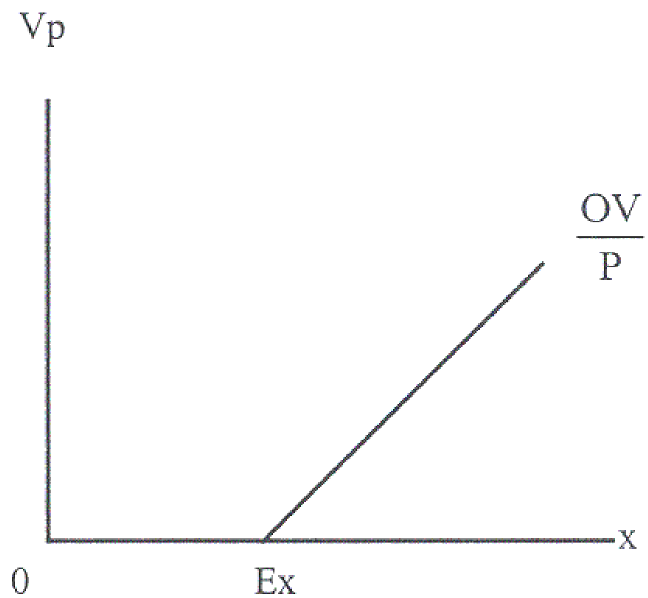

Figure 1: The Value of Option and the Exercise Price

property and liability insurers' financial solvency is more affected by the underwriting risk rather than by the investment risk. That is true because the loss distribution of many property liability insurance lines are more uncertain. For instance, a big earthquake or a hurricane could cost an insurer severe losses that may run into the billion of dollars. Furthermore, since most lines of property liability insurance are short-tailed, to match potential claim payments, property liability insurers are forced to invest most of their money in short-term financial assets. In contrast, most life insurers are heavily invested in long-term financial assets. As a result, property-liability insurers' insolvency are less directly affected by financial market conditions. Brown and Hoyt [1995] find that the market interest rate and unexpected inflation rate do not significantly affect the frequency of insolvency in the property liability insurance industry. But, the number of insurers operating in the market and the combined ratio, which is equal to the sum of the loss ratio and the expense ratio, have significant effects on insolvency.

The conclusion that can be reached from the above discussion is that when one considers the market value of insolvency for property liability insurers, one may assume that the investment return is approximately constant. Consequently, the insurer's total value $(1+k)(1+r_i)$ will be the exercise price on the option.

Figure 1 describes the relation among the loss ratio, the exercise price and the option value, where $Ex = (1+k)(1+r_i)$ is the exercise price, x is the loss ratio, Vp is the market value of the option.

Since the market value of an option (either call or put) is convex at its exercise price [Huang and Litzenberger, 1988, p. 161], one expects that the second derivative of Vp with respect to k to be positive. As a result, the relation between the option value Vp and the policyholder's surplus ratio k is an inverse relationship. When the surplus ratio is increasing, the option value is decreasing and goes to zero.

To test the effects of the insolvency risk on the insurers' underwriting profit rate, one needs to assume some functional form of the market value of the option. A simple function such as $Vp = a\exp(-k)$ or $Vp = a/k$, where a is constant and non-negative, satisfies the features of the option discussed above. Then, one can estimate (8) by imposing the relevant condition of the option value, along with δ and c as defined before, in the equation. As a result, one has:

$$E(r_u) = -\delta(1-c)r_f + \beta_u(E(r_m) - r_f) - a\exp(-k) \tag{8a}$$

for $Vp = a\exp(-k)$; and

$$E(r_u) = -\delta(1-c)r_f + \beta u(E(r_m) - r_f) - a/k \tag{8b}$$

for $Vp = a/k$.

Estimating Underwriting Beta and the Effect of Capital Default Risk on the Profit Rate

In this section, the econometric model developed in the previous section is used to test whether the insolvency risk of insurers significantly affects the underwriting profit rate.

Econometric Models

From (8a) and (8b), one has the following regression models:

$$r_u = \alpha + \beta r_m + \gamma\exp(-k) + \varepsilon \tag{17}$$

for $Vp = a\exp(-k)$; or

$$r_u = \alpha + \beta r_m + \gamma/k + \varepsilon \tag{18}$$

for $Vp = a/k$; where r_u is the underwriting profit rate; r_m is the rate of return on the market portfolio; k is the policyholder's surplus ratio; and ε is a mean-zero disturbance term.

By Relating (17) to (8a) and (18) to (8b), one has $\gamma = -a$, and $\beta = \beta_u$. The null hypothesis is $\gamma = 0$. This implies that the possibility of insolvency does not affect the insurance rate. The alternative hypothesis is $\gamma < 0$.

In (17) and (18), an intercept is included. Comparing (17) with (8a) or (18) with (8b), one sees that the intercept contains a risk-free rate and underwriting costs.

The Data

The data on stock insurance companies from 1943-99 is used in the estimation. Other type of insurance companies such as mutual companies are not chosen due to lack of pertinent data. For example, A. M. Best Company does not report information about policyholder's surplus ratios for non-stock companies. The data of stock insurers ought to be more reliable for the estimation because these companies shares are publicly traded and, thereby, subject to more financial scrutiny by investors and the SEC.[5]

The data source is *Best's Aggregates and Averages* (various years). The aggregate data of stock insurers is used in the study. In other words, the total surplus and total net premiums written belonging to all stock insurers are used to calculate the policyholder's surplus ratio.

Here, one needs to interpret the meaning of the probability of insolvency for the whole industry. Since the probability of surviving for the whole industry is always 1, a positive probability of insolvency is meaningless. However, one may interpret the probability of insolvency as the average frequency of insolvency. Such a frequency of insolvency is relevant because it indicates the chance that policyholders' claims will be defaulted.

The S&P 500 common stocks are chosen as the market portfolio. The data for the rates of return for the S&P 500 is from Ibbotson's yearbook, *Stocks, Bonds, Bills, and Inflation*, published by Ibbotson Company. It is in nominal terms.

Stock Insurers Versus Mutual Insurers

A stock insurer is an insurance firm that is owned by stockholders. The other major type of insurers are mutual companies, which are owned by their policyholders. In the United States, the majority of insurance firms are owned by stockholders. The advantage of being a stock insurer is the easy access to the financial market. Such an advantage enables the insurer to raise needed capital and to expand its underwriting capacity on a timely basis to meet an increase in the demand for insurance coverage. But stock insurers face more agency problems than mutual insurers because of the conflict of interests between policyholders and shareholders [Mayers and Smith, 1981; 1986; 1988; Fama and Jensen, 1983a; 1983b). In addition, managers of stock insurers have more incentive to be involved in risky insurance activities than the managers of mutual insurers because their compensations are usually higher and more responsive to their firms' performances [Mayers and Smith, 1992; Tennant and Starks, 1993]. As a result, stock insurers will likely be more associated with the riskier lines of business and geographical areas. Empirical studies support such a prediction [Mayers and Smith, 1988; 1992; Tennant and Starks, 1993]. Consequently, stock insurers will have a higher risk of being insolvent. In the property and casualty insurance market, there are more stock companies than there are mutual companies. The ratio is about seven to three. Also, historically, the frequency of insolvencies is much higher among stock companies than among mutual companies. This ratio is about 8:2.

Estimated results

Three different models are estimated. In Model 1, the value of insolvency option has the form of $Vp = a\exp(-k)$, and in Model 2 it is assumed to be $Vp = a/k$. In addition, the traditional CAPM without considering the default risk (called Model 3) is estimated to be comparable with the other two models. As a result, one has the following three models:

$$\begin{aligned} &\text{Model 1} \quad : \quad r_u = \alpha_1 + \beta_1(S\&P500) + \gamma_1\exp(-k) + \varepsilon_1 \\ &\text{Model 2} \quad : \quad r_u = \alpha_2 + \beta_2(S\&P500) + \gamma_2(1/k) + \varepsilon_2 \\ &\text{Model 3} \quad : \quad r_u = \alpha_3 + \beta_3(S\&P500) + \varepsilon_3 \quad , \end{aligned}$$

where r_u is the underwriting profit rate, $(S\&P500)$ is the rate of return on the S&P 500, k is the policyholder's surplus ratio. ε_1, ε_2, and ε_3, are error terms, and α_1, α_2, α_3, β_1, β_2, β_3, γ_1, and γ_2 are parameters to be estimated.

Simple OLS Estimation

Table 1 reports the estimated results from the ordinary least square (OLS) estimation. The table shows that the estimated coefficient of the variable $\exp(-k)$ is significant at the 5 percent level in Model 1 and that the coefficient of the variable $(1/k)$ is significant at the 5 percent level in Model 2. The sign of both coefficients is negative and so the existence of insolvency risk significantly reduces the underwriting profit rate. The estimated underwriting beta in all three models is small and not significant; it is negative in Models 1 and 2 but positive in Model 3. The absolute value of the beta is larger in Models 1 and 2 than in Model 3. The conclusion is that the estimated underwriting beta is sensitive to the choice of estimation methods. A negative beta implies that the insurers' loss ratio is positively related to the economy; as a result, the insurers' underwriting profit rate is negatively related to the economy. Previous studies have found that the beta could be either positive or negative. For instance, Cummins and Harrington [1985] determine the beta to be negative based on profit rate, but Fairley [1979] and Hill [1979] determine it to be positive.

TABLE 1
Estimated Results for Stock Insurance Companies, 1943-99 (Simple OLS)

Variable	Model 1 Estimate	Model 2 Estimate	Model 3 Estimate
Constant	0.0914	0.0623	-0.021
	(0.0528)	(0.0420)	(0.120)
$S\&P500$	-0.0291	-0.0271	0.009
	(0.0531)	(0.0534)	(0.050)
$\exp(-k)$	-0.2350		
	(0.1085)**		
$(1/k)$		-0.0597	
		(0.0292)**	
Number of Observations	57	57	57
R^2	0.09	0.08	0.00
Adjusted R^2	0.05	0.04	-0.02
F-statistic	2.38	2.11	0.03

Note: Dependant variable is the underwriting profit rate (r_u). All models are estimated using simple OLS without considering the error term's order. Parenthesis indicate standard deviations. ** is significant at 5 percent. r_u is the underwriting profit (or loss) divided by the premium earned, where underwriting profit (or loss) is equal to the total premiums written reduced by losses and expenditures. That is, $r_u = 1-$ loss ratio $-$ expenditure ratio, with the loss ratio equal to total losses and loss adjusted expenses incurred divided by the premium earned, and the expenditure ratio equal to the total underwriting expenditures divided by the premium earned. k is the policyholder's surplus ratio, which equals the policyholder's surplus divided by the net premium written, with policyholder's surplus equal to the total assets $-$ liabilities.

The adjusted R^2 and F-statistic are 0.05 and 2.38, respectively, from Model 1, 0.04 and 2.11 from Model 2, and -0.02 and 0.03 from Model 3. These numbers indicate that Models 1 and 2 fit the data better than Model 3, but that none of the F-statistics in the three models is significant at the 5 percent level; a finding which suggests that one needs to adjust the assumption on the error term in the models.

OLS with AR (2) Estimation

Table 2 reports the estimated results when the error terms in the three models are imposed to be $AR(2)$. The error term's process is necessary because the circumstances of the insurance market varied considerably during the time period from 1943-99. One would reasonably expect that the error term in the estimation equations will be heteroskedastic and serially correlated. Especially, the cyclical nature of the insurance market will cause the disturbance to be serially correlated. In addition, measurement problems in accounting (time lags in filing and settling claims) and the insurer's strategy to smooth its profit curve [Cummins and Harrington, 1985] can also cause the disturbance to be serially correlated. Therefore, one needs to re-estimate the model considering the error term's process. By analyzing each model's residuals from the simple OLS, it turns out that the error term in each model has $AR(2)$.[6] Then the models are re-estimated using the OLS method under the assumption that the disturbance terms have $AR(2)$.

Table 2 shows that when the error term is imposed to be $AR(2)$, most variables' significance does not change, except for the variable $(1/k)$ which is significant at the 5 percent level in the simple OLS, but not significant when $AR(2)$ is used in the model. In addition, the signs of all of the estimated coefficients do not change. The fact that the coefficient of $\exp(-k)$ is -0.3117, which is larger in absolute value than before, implies that the insolvency risk significantly reduces the required insurance rate more in the model with the $AR(2)$ error term than before. The major benefits by imposing $AR(2)$ on the error term are that the adjusted-R^2 and the F-statistic are improved dramatically. In all three models, the adjusted R^2 is 0.70 or bigger. In particular, the F-statistic becomes significant at the 1 percent level in all three models. In other words, when the error term is considered to be $AR(2)$, all three models fit the data very well.

TABLE 2
Estimated Results for Stock Insurance Companies
by Imposing $AR(2)$ on the Error Term, 1943-99

Variable	Model 1 Estimate	Model 2 Estimate	Model 3 Estimate
Constant	0.1258	0.0569	-0.0236
	(0.0709)	(0.0516)	(0.0167)
$S\&P500$	-0.0321	-0.0205	0.0026
	(0.0224)	(0.0213)	(0.0168)
$\exp(-k)$	-0.3117		
	(0.1464)**		
$(1/k)$		-0.0579	
		(0.0355)	
Number of Observations	57	57	57
R^2	0.75	0.74	0.72
Adjusted R^2	0.72	0.71	0.70
F-statistic	30.73*	29.23*	36.65*

Note: Dependant variable is the underwriting profit rate (r_u). All models are estimated by imposing $AR(2)$ on the error term. Parenthesis are standard deviations. * is significant at the 1 percent level and ** is significant at 5 percent. The definition of variables is the same as in Table 1.

The estimated underwriting beta in all three models are still not significant; a finding consistent with the simple OLS estimates of beta reported in Table 1. In other words, the change of the rate of return of the market portfolio does not significantly affect insurers' underwriting profit. One interpretation, mentioned earlier, is that property-liability insurers are less influenced by the market interest rate because most of their policies are of a short-term nature, which is different from life insurers that issue long-term policies with their profits, thus, more closely linked to the market interest rate. However, economic and market conditions such as the market interest rate, the inflation rate, the number of insurers and the whole industry's loss ratio still indirectly affect insurers' underwriting profit rate because these factors influence the insurers' market values and the probabilities of their insolvency.

Robustness of the Estimation

In the above estimations, the rate of return of the S&P 500 is nominal, not adjusted by the relevant inflation rate. When the real rate of return of the S&P 500 is used, the estimated signs and significance of the coefficients for all the estimates are not changed. In addition, when the equity risk premium instead of the rate of return of the S&P 500 is used, the results are not significantly different either.

Underwriting Profit Rate

Using the models discussed above, one can predict the underwriting profit rate associated with each model. Table 3 reports the results for the simple OLS estimation. This table shows that the actual average profit rates for stock insurance companies during 1943-99 was -1.935 percent with a standard deviation of 5.95 percent. Without considering the error term's serial correlation, all three models have the same fitted average value of -1.935 percent as the actual one. But for Model 1, the standard deviation of the fitted value is closer to the actual one. On the other hand, the traditional CAPM (Model 3) has the biggest standard deviation of the residuals which implies that Model 3 is less reliable than the other models.

TABLE 3
Actual and Fitted Values of Underwriting Profit Rates
from Simple OLS Estimation, 1943-99

	Mean (Percentage)	Standard Deviation (Percentage)
Actual	-1.935	5.95
Model 1 Fitted	-1.935	1.83
Model 1 Residual	0.000	5.67
Model 2 Fitted	-1.935	1.72
Model 2 Residual	0.000	5.70
Model 3 Fitted	-1.935	0.16
Model 3 Residual	0.000	5.98

Note: Model 1, 2, and 3 are the same as in Table 1 and are estimated without considering the error term's process. Fitted and residuals are the average values of fitted and residuals. Residual = Actual - Fitted.

For the estimation using the OLS with $AR(2)$, the results, which are shown in Table 4, indicate that the actual average profit rate during 1945-99 was -2.25 percent with a standard deviation of 5.86 percent. When the error term is considered to be $AR(2)$, all models have the same average fitted value as the actual one, but Model 3 has the largest standard deviation of residuals and so it is again less reliable than the other models.

TABLE 4
Actual and Fitted Values of Underwriting Profit Rates
with $AR(2)$ Error Term, 1943-99

	Mean (Percentage)	Standard Deviation (Percentage)
Actual	-2.25	5.86
Model 1 Fitted	-2.25	2.71
Model 1 Residual	0.00	2.96
Model 2 Fitted	-2.25	2.30
Model 2 Residual	0.00	3.01
Model 3 Fitted	-2.25	1.68
Model 3 Residual	0.00	3.12

Note: Models 1, 2, and 3 are the same as in Table 2 and are estimated by imposing $ARMA(2,2)$ on the error term. The fitted and residual are average values of fitted and residuals. Residual = Actual - Fitted.

One conclusion stemming from the above results is that in the CAPM considering the error term's process is very important. One explanation is that insurers adjust their insurance premiums each year based on their past experience of insurance losses. As a result, the insurers' underwriting profit rate is serially correlated. The other explanation is that insurers can not change their insurance rate dramatically because of rate regulation. In order to change insurance rates, insurers in many states need to file requests with their states' insurance departments which usually require them to verify the reasons as to why the rates need to be changed.

Conclusions and Implications

The Capital Asset Pricing Model (CAPM) has been widely used to derive a fair price of insurance. But the traditional CAPM over-estimates insurance premiums because it does not include the insolvency risk of insurers. This paper examines how the insurance price should be fairly adjusted when insurers have a default risk. This study further confirms that insurance premiums should be lower when insurers have a positive probability of being insolvent. The study further shows that previous estimates of the underwriting beta without considering the default risk are biased.

Using data of property liability stock insurers from 1943-99, the paper further estimates the underwriting beta and the effects of insolvency risk on pricing insurance. It shows that the insolvency risk significantly reduces the price of insurance. Based on the estimated coefficient from the model the estimated total effect of the insolvency risk on the property-liability insurers' underwriting profit rate is about 0.14 percent which is close to the ratio of premium volume of insolvencies to the total volume of premiums for the property-liability insurance industry. Although the total effect on insurers' profit rate is small, the associated effect on insurers' total profits are very large. During the period examined, the average volume of premiums written by property liability stock insurers was $48 billion per year. If the default risk were to be included in the model, the calculated premium would have been lower and this would have in effect reduced profit, by about $67 million.

Footnotes

[1]The rate of return of underwriting equals (1- the loss ratio - the expenses ratio), where the loss ratio is defined by the expected loss divided by the premiums and the expenses ratio equals the total underwriting expenses divided by total premiums.

[2]In property liability insurance, during 1969 and 1983, the average number of insolvency was 9.8 per year and frequency of insolvency was only 0.3 percent; however, during 1984-98, these numbers were 30 and 1.22 percent [Best's Review, Property/Casualty Edition, March 1999, pp. 55-67].

[3]Ibid.

[4]The put option is negatively correlated to the market interest rate [Huang and Litzenberger, 1988].

[5]All stock companies are closely monitored by public investors. Any trouble a stock company encounters will cause its stock price to change. Also, stockholders are more aware of and sensitive to the insolvency risk of their company than owners of other types of insurance companies, such as mutual insurers.

[6]The LR (Log-likelihood Ratio) Test shows that one does not reject $ARMA(2,2)$.

References

A. M. Best Company. *Best's Aggregates and Averages, Property and Liability Edition*, Oldwick, NJ (various years).

—. *Best's Insolvency Study*, Oldwick, New Jersey, 1991.

Biger, N.; Kahane, Y. "Risk Considerations in Insurance Rate-Making," *Journal of Risk and Insurance*, 45, 1978, pp. 121-32.

Bronars, S. G. "Fair Pricing of Unemployment Insurance Premiums," *Journal of Business*, 58, 1, 1985, pp. 27-47.

Brown, M. J.; Hoyt, R. E. "Economic and Market Predictors of Insolvencies in the Property-Liability Insurance Industry," *Journal of Risk and Insurance*, 62, 1995, pp. 309-27.

Chen, Y.; Hamwi, I. ; Hudson, T. "The Effect of Ceded Reinsurance on Solvency of Primary Insurers," *International Advances in Economic Research*, 7, 1, February 2001, pp. 65-82.

Cummins, J. D.; Harringston, S. "Property-Liability Insurance Rate Regulation: Estimation of Underwriting Betas Using Quarterly Profit Data," *Journal of Risk and Insurance*, 52, 1, 1985, pp. 16-43.

D'Arcy, S. P.; Doherty, N. A. *The Financial Theory of Pricing Property-Liability Insurance Contracts*, S. S. Huebner Foundation for Insurance Education, University of Pennsylvania, 1988.

Derrig, R. "The Use of Investment Income in Massachusetts Private Passenger Automobile and Workers' Compensation Ratemaking," in J. D. Cummins and S. Harrington, eds., *Fair Rate of Return in Property-Liability Insurance*, Hingham, MA: Kluwer-Nijhoff, 1986.

Doherty, N. A.; Garven, J. R. "Price Regulation in Property Liability Insurance: A Contingent Claims Approach," *Journal of Finance*, 40, 1986, pp. 1031-50.

Fairley, W. B. "Investment Income and Profit Margins in Property-Liability Insurance: Theory and Empirical Results," *Bell Journal of Economics*, 10, 1979, pp. 192-210.

Fama, E.; Jensen, M. "Separation of Ownership and Control," *Journal of Law and Economics*, 26, 1983a, pp. 301-25.

—. "Agency Problems and Residual Claims," *Journal of Law and Economics*, 26, 1983b, pp. 327-49.

Hill, R. D. "Profit Regulation in Property-Liability Insurance," *Bell Journal of Economics*, 10, 1979, pp. 172-91.

Hill, R.; Modligliani, F. "The Massachusetts Model of Profit Regulation in Non-Life Insurance: An Appraisal and Extensions," in J. D. Cummins and S. Harrington, eds., *Fair Rate of Return in Property-Liability Insurance*, Hingham, MA: Kluwer-Nijhoff, 1986.

Huang, C. F.; Litzeuberger, R. H. *Foundations for Financial Economics*, New York: North Hotland, 1988.

Ibbotson, R. G. "Stocks, Bonds, Bills, and Inflation," *Yearbook* (various years).

Lee, Soon-Jae. "Solvency Regulation in the Property-liability Insurance Industry: Re-examination," Paper presented at the annual meeting of the American Risk and Insurance Association, Toronto, Canada, August 1994.

Litner, J. "Security Prices, Risk and Maximal Gains from Diversification," *Journal of Finance*, 20, 1965, pp. 587-615.

Mossin, J. "Equilibrium in a Capital Asset Market," *Econometrica*, 34, 1966, pp. 768-83.

Munch, P.; Smallwood, D. "Solvency Regulation in the Property-Liability Insurance Industry: Empirical Evidence," *Bell Journal of Economics*, 11, 1980, pp. 261-83.

Mayers, D.; Smith, C. W. "Executive Compensation in the LifeInsurance Industry," *Journal of Business*, 65, 1992, pp. 51-74.

—. "Ownership Structure Across Lines of Property-Liability Insuarnce," *Journal of Law and Economics*, 31, 1988, pp. 351-78.

—. "Ownership Structure and Control: Mutualization of Stock Life Insurance Companies," *Journal of Financial Economics*, 16, 1986, pp. 73-99.

—. "Contractual Provisions, Organizational Structure, and Conflict Control in Insurance Markets," *Journal of Business*, 54, 3, 1981, pp. 407-34.

Sharpe, W. F. "Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk," *Journal of Finance*, 19, 1964, pp. 74-80.

Tennant, J. L.; Starks, L. T. "Stock Versus Mutual Ownership Structures: The Risk Implications," *Journal of Business*, 66, 1993, pp. 29-46.

U.S. Congress (House), Energy and Commerce Committee, "Failed Promises: Insurance Company Insolvencies," Report by the Subcommittee on Oversight and Investigations, 1999.

Winter, Ralph A. "Solvency Regulation and the Property-Liability Insurance Cycle," *Economics Inquiry*, 29, 1992, pp. 458-71.