Solvency Regulation in the Property-Liability Insurance Industry

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Abstract

This paper evaluates guaranty funds and solvency regulations. One main question addressed is how solvency regulations will benefit consumers. Many previous studies have found that most forms of solvency regulations do not have significant deterrent effects on insolvency. Even when solvency regulations are effective, they might still adversely affect consumers. This could happen because increasing the probability of solvency usually requires raising premiums. Therefore, it is interesting to see how regulators should design insurance regulations that benefit consumers. Insolvency of insurance firms provides a unique environment under which one is able to analyze the effects of solvency regulations and guaranty funds on the quality of insurance products and on consumers. This paper shows that guaranty funds are always desirable, but solvency regulations are of certain value only when they have the effect of protecting guaranty funds and alleviating the disincentives which they create. (JEL G22); *Int'l Advances in Econ. Res., 10(4): pp. 313-327, Nov. 04.*

Introduction

Because of the increase in insolvencies of insurance firms in recent years,[1] the effectiveness of insurance guaranty funds and solvency regulations[2] has been questioned [Jackson, 1990; U.S. House, 1990; Power, et. al., 1991; Schacht and Gallanis, 1993]. Some proposals have been put forward to reform the current regulatory system. One suggestion is to have more interstate cooperation and to form interstate compacts of guaranty funds [Jackson, 1990; Schacht and Gallanis, 1993]. The other is to have more federal intervention, which includes setting up a national guaranty fund similar to the federal deposit insurance system and regulating insurance firms through the federal government [U.S. House, 1990]. However, the necessity and effectiveness of such reforms, and particularly that of federal intervention, have been doubted.

This paper evaluates guaranty funds and solvency regulations. One main question addressed is how solvency regulations will benefit consumers. Many previous studies have found that most forms of solvency regulations do not have significant deterrent effects on insolvency [Munch and Smallwood, 1980; Lee, 1994]. Even when solvency regulations are effective, they might still hurt some consumers [Doherty and Schlesinger, 1990]. This could happen because increasing the probability of solvency usually requires raising premiums. Therefore, it is interesting to see how regulators should design insurance regulations to benefit consumers.

This study can be viewed as an extension of the studies of regulatory effects on quality of products and on consumers. Here the quality of insurance products is measured by the firm's probability of solvency and its ability to pay claims. Economists have long been

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interested in such studies. Spence [1975] finds that when firms have the monopoly power to set both the price and quality of products, they tend to set quality too low. He further concludes that rate of return regulation will force firms to raise quality and thereby benefit consumers, provided quality is a capital-using attribute. Many previous studies have focused on the effects of regulations on the quality of insurance services, instead of the quality of the product itself. Findings from these studies have been controversial. For instance, Frech and Samprone [1980] find that rate regulation in property-liability insurance leads to non-price competition, i.e. competition centering on the quality of services. However, Pauly, Kunreuther, and Kleindorfer [1986] find that rate regulation reduces the average price of many types of insurance and lowers the quality of insurance services. They conclude that the welfare effects of such regulation are not clear and need to be explored further.[3]

Insolvency of insurance firms provides a unique environment under which one is able to analyze the effects of solvency regulations and guaranty funds on quality of insurance products and on consumers. This paper intends to derive theoretical results. More work, particularly empirical work, would be needed to add understanding to the subject.

Demand for Insurance

Theory suggests that a risk averse expected utility maximizer should be fully insured if the premium is fair and partially insured if there is a positive loading factor [Arrow, 1963; Mossin, 1968; Smith, 1968]. One assumption underpinning these results is that the insurance contract has no default risk. Many researchers have explored the demand for insurance when such an assumption is relaxed. For instance, Tapiero, et. al. [1986] examine how much a consumer is willing to pay for full coverage when his contract has a default risk, and used this information to obtain a pricing scheme for an insurance monopolist. Schlesinger and Schulenburg [1987] illustrate the effects of consumer risk aversion on the demand for insurance when the insurer has an insolvency risk. Doherty and Schlesinger [1990] explore the optimal demand in the presence of contract non-performance. They show that consumers will be partially insured at an actuarially fair price and that they may purchase less coverage as the insurer becomes more solvent.

The model used here is similar to that of Doherty and Schlesinger [1990], but it allows for the indemnity to be non-zero when the insurer is insolvent. That is, the insured will get partial recovery for the loss. In their model, such an indemnity is assumed to be zero. This modification makes the welfare analysis possible.

The Model

The consumer has initial wealth W and has a probability q of suffering a loss L. He is risk averse and an expected utility maximizer; i.e. he has Von Neumann-Morgenstern utility $U(\bullet)$ with $U'(\bullet) > 0$ and $U''(\bullet) < 0$.

The consumer is offered insurance with the premium π per dollar coverage ($0 \leq \pi \leq 1$). Then, for y dollars of coverage he must pay $\pi\, y (0 \leq y)$. The consumer knows that the insurer has a probability s of being solvent $(0 \leq s \leq 1)$; in other words, the probability of insolvency of the insurer is $(1 - s)$.[4] If the insurer becomes insolvent, the default indemnity will be θ per unit of coverage $(0 \leq \theta \leq 1)$.

Let state 2 be the one in which a loss occurs and the insurer is solvent, state 1 be the case where a loss occurs but the insurer is insolvent, and state 0 be the state of no loss. Then the consumer's contingent wealth in these three states will be:

$$W_2 = W - L - \pi y + y, W_1 = W - L - \pi y + \theta y, and W_0 = W - \pi y, \text{ respectively} \quad .$$

As a result, the consumer's optimization problem is:

$$\text{Max. } G(y) = qsU(W_2) + q(1-s)U(W_1) + (1-q)U(W_0) \quad , \tag{1}$$

$$y \text{ Subject to } 0 \leq y \leq W \quad .$$

The first order condition for problem (1) is $G'(y) = 0$, or:

$$qs(1-\pi)U'(W_2) + q(1-s)(\theta-\pi)U'(W_1) - (1-q)\pi U'(W_0) = 0 \quad . \tag{2}$$

And, the second order condition is satisfied because:

$$\begin{aligned} G''(y) &= qs(1-\pi)^2U''(W_2) + q(1-s)(\theta-\pi)^2U''(W_1) + (1-q)\pi^2U'' \\ (W_0) &< 0 \text{ due to } U''() < 0 \quad . \end{aligned}$$

In case $s = 1$ or $\theta = 1$, problem (1) is reduced to the standard one:

$$\text{Max. } G(y) = qU(W_2) + (1-q)U(W_0) \quad ,$$

$$\text{again } y \quad \text{Subject to } 0 \leq y \leq W \quad . \tag{1$'$}$$

So, the optimal insurance from problem (1$'$) will be $y^* = L$ for $\pi = q$; and $y* < L$ for $\pi > q$.

Let y^* be the optimal solution from problem (1), and for brevity, still denote W_2, W_1, and W_0 to be the consumer's contingent wealth at $y = y^*$, then the consumer's indirect utility is:

$$V^* = qsU(W_2) + q(1-s)U(W_1) + (1-q)U(W_0) \quad . \tag{3}$$

The consumer's demand for insurance will depend on many factors, such as q, s, θ, π, L, and W; so will his indirect utility. In other words, one has $y^* = y(q, s, \theta, \pi)$, and $V^* = V(q, s, \theta, \pi)$, where variables L and W are excluded in the equations for brevity. Note that $G(y)$ is the expected utility for the selected insurance coverage y. Hence, $G(L)$ is the value of G for $y = L$.

The Conditions for Full Insurance

To illustrate the conditions for full insurance, it is assumed here that the insurance market is perfectly competitive, and that there are no transaction costs. Thus, each insurance firm will have zero expected profits. The premium under such conditions is actuarially fair.

Since the expected net indemnities are $qs(1-\pi) + q(1-s)(\theta-\pi)$, and the expected income to the insurer at the no-loss state is $(1-q)\pi$, then under the actuarially fair premium assumption, one has:

$$qs(1-\pi) + q(1-s)(\theta-\pi) = (1-q)\pi \quad . \tag{4}$$

Consequently the actuarially fair premium would be:

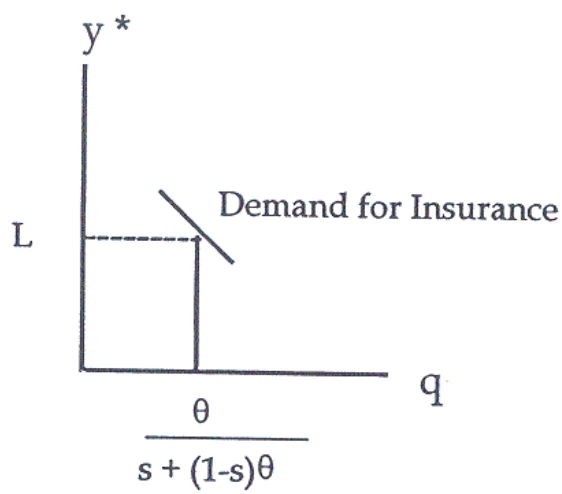
$$\pi = q(s + (1-s)\theta) \quad . \tag{5}$$

Equation (5) means that the insurance premium per dollar of coverage equals the expected payoff for claims. In the case of assured solvency, $s = 1$, the premium is equal to the expected loss, $\pi = q$. Clearly, $\pi \leq q$. Equation (5) shows that $\pi = q$ if and only, if $s = 1$ or $\theta = 1$. Thus, the fair premium is below the expected loss because there is a default risk.[5]

Proposition 1. Assuming an actuarially fair premium, consumers with risk $q < \frac{\theta}{s+(1-s)\theta}$ will be overly insured, consumers with risk $q > \frac{\theta}{s+(1-s)\theta}$ will be partially insured, and consumers with risk $q = \frac{\theta}{s+(1-s)\theta}$ will be fully insured.

The proof of this proposition follows. Notice that $G''(y) < 0$, thus $G'(y)$ is a decreasing function of y. Using condition (4), $qs(1-\pi) + q(1-s)(\theta-\pi) = (1-q)\pi$, it is easy to show that $G'(0) > 0$, which indicates that the consumer will have positive coverage. In addition, since $G'(L) = q(1-s)(\theta-\pi)[U'(W_1) - U'(W_0)], G'(L) \leq (\geq)\ 0$ only when $\theta \leq (\geq)\ \pi$ due to $U'(W_1) > U'(W_0)$. However, $G'(L) = 0$ is the condition for $y^* = L$. On the other hand, $G'(L) < 0$ will be the condition for $y^* < L$ and $G'(L) > 0$ will be the condition for $y^* > L$. Using the fact $\pi = q(s + (1-s)\theta)$, one has $y^* \leq (\geq) L$ when $q \geq (\ \leq\ )\frac{\theta}{s+(1-s)\theta}$.[6]

FIGURE 1
Over Insurance and Under Insurance with Fair Premiums



The previous proposition is illustrated in Figure 1. The figure shows that there is a neighborhood around $q = \frac{\theta}{s+(1-s)\theta}$, such that $\frac{\partial y^*}{\partial q} < 0$.[7] This is true because y^* is continuous with respect to q, and because from the proposition $\frac{\partial y^*}{\partial q} < 0$ holds at $q = \frac{\theta}{s+(1-s)\theta}$. In addition, all consumers with $q < \frac{\theta}{s+(1-s)\theta}$ will be over-insured, while consumers with $q > \frac{\theta}{s+(1-s)\theta}$ will be under-insured. Alternatively, as the probability of solvency (s) worsens, more consumers will be over-insured, but when the recovery fraction θ worsens, more consumers will be partially insured.[8]

The above result that a consumer with a higher risk will purchase less insurance than others, is quite surprising. To understand the result, look at state 1 in which the loss occurs and the insurer is insolvent. At a price of $\pi = \theta$, there is a break even point at which insureds are indifferent as to being with insurance or without insurance. At $\pi > \theta$, insureds are worse off with insurance; in contrast at $\pi < \theta$, insureds are better off. Since $\pi = q(s + (1-s)\theta)$, the condition $\pi < \theta$ becomes q $< \frac{\theta}{s+(1-s)\theta}$. Then consumers with lower risk would prefer to purchase more insurance than they would usually do (full insurance). They pay lower premiums with the existence of insolvency of the insurer. In case the insurer is insolvent, they are still better off with insurance than otherwise. With similar reasoning, one can understand the case of consumers with higher risk.

Properties of the Indirect Utility

The following properties of indirect utility are given here because they will be used in the proofs of some theorems later on.

Proposition 2. Assuming π is constant, then:

$$\frac{\partial V(q,s,\theta,\pi)}{\partial s} > 0, \ \frac{\partial V(q,s,\theta,\pi)}{\partial \theta} > 0, \text{ and } \frac{\partial V(q,s,\theta,\pi)}{\partial \pi} < 0$$

The proof of these relationships follows easily from the definition of the indirect utility given in equation (3). The above relationships simply say that the indirect utility is an increasing function of both the probability of solvency and the indemnity, other things equal, and the indirect utility is a decreasing function of the premium.

Changing Either the Default Indemnity or the Probability of Solvency

One is able to analyze the effectiveness of guaranty funds and solvency regulation by applying the demand and indirect utility functions given previously. However, two lemmas are first given.

Lemmas

To be more general, assume that there is a fixed loading factor r in the premium, i.e. $\pi = q(1+r)(s+(1-s)\theta)$ with $r = 0$. The loading factor is added to compensate for the insurer's transaction costs or to allow the insurer to earn a normal profit rate. In case there is no loading, one has $\pi = q(s+(1-s)\theta)$, which means the premium is actuarially fair.

The π formula shows that consumers will pay extra premiums when the default indemnity (θ) is increased or the probability of solvency(s) is increased. For instance, suppose that the default indemnity is increased by x percent through the state's guaranty fund, then a consumer with risk q will pay extra premiums of $q(1+r)(1-s)x$,[9] where $q(1-s)x$ is the expected payment from the fund. The loading factor is added to account for possible administration or other costs associated with the fund.

Lemma 1.

$$\begin{aligned} & qsU'(W_2) + q(1-s)\theta U'(W_1) \\ = \ & \pi[qsU'(W_2) + q(1-s)U'(W_1) + (1-q)U'(W_0)] \quad . \end{aligned} \tag{6}$$

Proof: Since y^* is the optimal solution from problem (1), y^* satisfies the first order condition (2), i.e.:

$$qs(1-\pi)U'(W_2) + q(1-s)(\theta-\pi)^*U'(W_1) - (1-q)\pi U'(W_0) = 0$$

Then, (6) is the direct result from the above equation.

Lemma 2.

$$\frac{\partial V^*}{\partial \theta} = q(1-s)U'(W_1)y^* - q(1+r)(1-s)y^*\frac{1}{\pi}[qsU'(W_2) + q(1-s)\theta U'(W_1)] \quad . \tag{7}$$

Proof: Using the envelope theorem and the definition of π, one has:

$$\frac{\partial V^*}{\partial \theta} = \partial V/\partial\theta + \partial V/\partial\pi \frac{\partial \pi}{\partial \theta}$$

$$\begin{aligned} &= q(1-s)U'(W_1)y^* - [qsU'(W_2) + q(1-s)U'(W_1) + (1-q)U'(W_0)]y^*q(1+r)(1-s) \\ &= q(1-s)U'(W_1)y^* - q(1+r)(1-s)y^*\frac{1}{\pi}[qsU'(W_2) + q(1-s)\theta U'(W_1)] \quad . \end{aligned} \tag{8}$$

The last equation is by Lemma 1.

Theorem 1.

Here the paper examines the welfare effects when only the indemnity or the probability of solvency is changed and has the following theorem, $\partial V^*/\partial\theta > 0$.

Proof: By Lemma 2, one has:

$$\begin{aligned} \partial V^*/\partial\theta &= q(1-s)U'(W_1)y^* - q(1+r)(1-s)y^*\frac{1}{\pi}[qsU'(W_2) + q(1-s)\theta U'(W_1)] \\ &> q(1-s)U'(W_1)y^* - q(1+r)(1-s)y^*\frac{1}{\pi}[qsU'(W_1) + q(1-s)\theta U'(W_1)] \quad (9) \\ &= q(1-s)y^*\frac{1}{\pi}[\pi - q(1+r)(s+(1-s)\theta)]U'(W_1) \quad . \quad (10) \end{aligned}$$

Thus, $\partial V^*/\partial\theta > 0$ due to $\pi = q(1+r)(s+(1-s)\theta)$. Equation (9) is correct by $U'(W_2) < U'(W_1)$, and equation (10) is correct because of $\pi = q(1+r)(s+(1-s)\theta)$.

Since the objective of states' guaranty funds is to raise the default indemnity,[10] Theorem 1 implies that they are certain to benefit consumers. Furthermore, since a consumer's indirect utility is an increasing function of the default indemnity, consumers will achieve their maximum welfare or the maximum indirect utility when their claims are fully guaranteed. However, such a conclusion is based on the condition that the premiums are fair. In other words, when the premiums are not fair, consumers will prefer sharing some default risk.

When premiums are fair, an increase in the default indemnity always raises consumer welfare, but an increase in the probability of solvency, which may be due to an increase in premium, may lower consumer welfare. To ensure that consumers are better off when the probability of solvency is increased, one needs to impose more restrictions, such as the probability of solvency goes to 1, the default indemnity goes to 0, or that consumers have mean-variance preferences (see proofs in the Appendix).

Counter-Example (Constant Absolute Risk Aversion)

The following is an example in which the consumer's indirect utility is decreased when the probability of solvency is increased. Let $U(W) = 1 - 20\ \mathrm{Exp}(-W)$, $q = \theta = .3$, $L = W = 20$, and $r = 0$, Then, since $q \leq \frac{\theta}{s+(1-s)\theta}$ for all s, from Proposition 1 one has $y^* \geq L$ for all s. Here, $y^* > L$ is excluded due to the fact that in reality one does not observe insurance coverage being larger than the maximum possible loss or even larger than total wealth. Thus, the corner solution $y^* = L$ is maximum for all s. Furthermore, the calculation of the first order condition at $y^* = L$ shows that $G'(L)$ has been near zero.

Table 1 gives the consumer's indirect utility V^* and equivalent wealth loss W_e at different levels of solvency. The equivalent wealth loss, which is the amount of wealth the consumer is willing to give up to avoid uncertainty, is calculated by solving $V(q, s, \theta, \pi, W) = V(q, 1, \theta, q, W - W_e)$. The table shows that the indirect utility is decreasing and that the wealth loss is increasing in the range of s from 0 to 0.8.

TABLE 1
Counter-Example

s	π	y^*	$G'(y^*)$	V^*	W_e
0	.09	20	.00094	.91003	8.60161
.1	.111	20	.00116	.87676	8.91183
.2	.132	20	.00140	.83327	9.21884
.3	.153	20	.00163	.77796	9.49884
.4	.174	20	.00183	.71034	9.76618
.5	.195	20	.00193	.63263	10.0027
.6	.216	20	.00188	.55270	10.19921
.7	.237	20	.00161	.48942	10.33196
.8	.258	20	.00109	.48194	10.34646
.9	.279	20	.00041	.60576	10.07249
.91	.2811	20	.00035	.62996	10.00966
.92	.2832	20	.00029	.65697	9.93321
.93	.2853	20	.00023	.68697	9.84292
.94	.2874	20	.00018	.72018	9.73240
.95	.2895	20	.00013	.75681	9.59057
.96	.2916	20	.00008	.79710	9.41082
.97	.2937	20	.00005	.84129	9.16388
.98	.2958	20	.00002	.88965	8.80411
.99	.2979	20	.00000	.94245	8.15438
1	.3	20	0	.99998	0

Note: (1) $U(W) = 1 - 20e^{-W}$, $W = L = 20, \theta = .3$, $q = .3$; and $\pi = q(s + (1-s)\theta)$. (2) Since $\theta \geq \pi$ for all $0 \leq s \leq 1$, from Proposition 1, one has $y^* \geq L$ for all s. However, by checking the first order condition (2) at $y^* = L$, one finds that $G'(L)$ is very close to zero for all $0 \leq s \leq 1$ (the biggest one is $G'(L) = .00193$ at $s = .5$). Thus, $y^* = L$ can be considered as the optimal solution for all s. (3) Let $V^* = V(q, s, \theta, \pi, W)$ be the indirect utility, then the wealth loss, denoted by W_e, is calculated by solving the following equation: $V(q, s, \theta, \pi, W) = V(q, 1, \theta, q, W - W_e)$ for each s; where $q = \theta = .3, W = 20$, and $\pi = q(s + (1-s)\theta)$.

Since solvency regulation is used to prevent insolvency,[11] the above counter-example indicates that such regulation may hurt some consumers. It is surprising that an increased probability of solvency might be harmful. One would think that a decrease in this probability would be beneficial. Such a conclusion, nevertheless, is wrong. Using Proposition 2, it can be easily shown that consumers will be worse off when the probability of solvency is decreased but the premium is increased.

It is possible that some forms of solvency regulations might lower the probability of solvency but raise premiums. First of all, an insurer may charge a higher premium to counter its extra costs associated with regulation. Secondly, regulators may wrongly identify an insurer as insolvent while in fact, the insurer can be financially solvent later on. Thus, regulation may at times lead to higher premiums and higher frequency of insolvency, thereby, adversely affecting consumers' welfare.

Changing Both the Indemnity and the Probability of Solvency

The results discussed previously are based on the assumption that either the indemnity or the probability of solvency is changed. When such an assumption is relaxed, some results will be different.

Both Indemnities and the Probability of Solvency are Increased

When the indemnity and the probability of solvency are increased, one has:

Theorem 2. Under fair premiums, $V(q, s+\Delta s, \theta+\Delta\theta, \pi+\Delta\pi) - V(q, s, \theta, \pi) > 0$ for Δs and $\Delta\theta > 0$.[12]

Proof:

$$V(q, s+\Delta s, \theta+\Delta\theta, \pi+\Delta\pi) - V(q, s, \theta, \pi) = V(q, s+\Delta s, \theta+\Delta\theta, \pi+\Delta\pi)$$
$$-V(q, s+\Delta s, \theta, \pi) + V(q, s+\Delta s, \theta, \pi) - V(q, s, \theta, \pi) \quad .$$

From Theorem 1, one has:

$$V(q, s+\Delta s, \theta+\Delta\theta, \pi+\Delta\pi) - V(q, s+\Delta s, \theta, \pi) > 0 \text{ for } \Delta\theta > 0 \quad ,$$

and from Proposition 2, one has:

$$V(q, s+\Delta s, \theta, \pi) - V(q, s, \theta, \pi) > 0 \text{ for } \Delta s > 0 \quad .$$

So $V(q, s+\Delta s, \theta+\Delta\theta, \pi+\Delta\pi) - V(q, s, \theta, \pi) > 0$ for $\Delta\theta$ and $\Delta s > 0$.

The above theorem says that under fair premiums, consumers will always be better off when the indemnity and the probability of solvency are increased. Guaranty funds and some forms of solvency regulation, therefore, will benefit consumers when they are able to prevent failures or reduce the costs of such failures.

Besides their role in protecting consumers, guaranty funds are also designed to prevent insolvency. Having such funds will raise consumers' confidence in the insurance industry and, thus, lower the possibility that some insurers would become insolvent because of consumers' fears. Secondly, since each solvent insurer shares the cost of insolvencies by paying its assessment to the state's guaranty fund, each insurer will, thus, have an incentive to monitor other insurers and to pressure state's regulators to enforce constraints on high risk behaviors [Munch and Smallwood, 1980]. Such voluntary and intensive monitoring will have preventive effects on failures of insurers. In this case, guaranty funds will be able to raise both indemnities and the probability of solvency, and therefore, as Theorem 2 suggests, they will benefit consumers.

Some forms of solvency regulation also play a role similar to that of guaranty funds. For instance, Lee [1994] shows that minimum capital and surplus requirements significantly reduce the frequency and costs of insolvencies. As a result, such forms of regulation will unambiguously benefit consumers. In addition, some forms of solvency regulation are able to protect guaranty funds by solving or alleviating agency problems, thereby benefitting consumers.

The agency problems will lead a firm to be more risk-taking [Jensen and Meckling, 1976; Shavel, 1979; Mayers and Smith, 1981]. Particularly, when firms are near being bankrupt, they will be more likely to adopt risky strategies.[13] By imposing solvency regulations, such as restrictions on investment and inspection of insurers' financial situation,[14] such problems will likely be prevented or at least they will be alleviated. As a result, both the frequency and costs of insolvencies will be lower. Thus, these forms of solvency regulation will benefit consumers.

The Premium is Fixed, but the Indemnity and the Probability of Solvency are Changed

Previously, the paper considered the welfare effects when premiums are changed along with changes in the indemnity and in the probability of solvency. Here, the paper considers such effects when the premium is fixed but the indemnity and the probability of solvency are changed.

Theorem 3. When π is constant, then $\frac{\partial V^*}{\partial s} \leq 0$ and $\frac{\partial V^*}{\partial \theta} \geq 0$.[15]
Proof: Since $\pi = q(1+r)(s+(1-s)\theta)$, by letting $\pi =$ constant, one will have:

$$(1-\theta)ds + (1-s)d\theta = 0 \quad . \tag{11}$$

Using the envelope theorem, one has:

$$\begin{aligned}\frac{\partial V^*}{\partial s} \quad | \quad {}_{\pi=const.} &= q[U(W_2) - U(W_1)] + q(1-s)U'(W_1)y^*(d\theta/ds) \\ &= q[U(W_2) - U(W_1)] + q(1-s)U'(W_1)y^*(-\frac{1-\theta}{1-s}) \quad ,\end{aligned} \tag{12}$$

$$= q[U(W_2) - U(W_1)] - q(1-\theta)U'(W_1)y^* \quad . \tag{13}$$

However, from the middle-value theorem, one knows:

$$U(W_2) - U(W_1) = U'(W^*)(W_2 - W_1) = U'(W^*)(1-\theta)y^* \quad , \tag{14}$$

where $W_1 \leq W^* \leq W_2$. Combining (13) and (14), one has:

$$\frac{\partial V^*}{\partial s} \mid_{\pi=const.} = q(1-\theta)y^*[U'(W^*) - U'(W_1)] \leq 0 \quad . \tag{15}$$

Equation (15) is due to $U'(W^*) \leq U'(W_1)$ for $W_1 \leq W^*$. Therefore, $\frac{\partial V^*}{\partial s} \mid_{\pi=const.} \leq 0$. $\frac{\partial V^*}{\partial \theta} \mid_{\pi=const.} \geq 0$ is obvious due to $\frac{\partial V^*}{\partial \theta} \mid_{\pi=const.} = \frac{\partial V^*}{\partial s} \mid_{\pi=const.*} (ds/d\theta) \mid_{\pi=const.}$.

Suppose that there are two types of guaranty funds or solvency regulation. Their costs are the same. Assume further that the first type will lead to having a higher indemnity, while the second type will lead to having a higher probability of solvency. Then, Theorem 3 says that it is in the consumer's interest to implement the first type of funds or to adopt the first type of regulation instead of using the second type.

Like federal deposit insurance,[16] states' guaranty funds have side effects and they may adversely affect firms' solvency. When consumers are guaranteed by a fund like the federal deposit insurance fund or a state guaranty fund, they tend to have less incentive to discipline firms' risk-taking behavior. In contrast, they may encourage the development of high risk and low solvent firms [Kane, 1989; White, 1989]. Because of consumers' reduced concern for firms' credibility, the firms will tend to be more risk-taking [Clair, 1984; Barth, et. al., 1989; Grossman, 1992]. In this case, guaranty funds will raise indemnities, but they might also raise the frequency of failures. From Theorem 3, one knows that such funds will benefit consumers as long as they do not raise premiums.

Similarly, some forms of solvency regulation would lead to lower costs of insolvency, but they may, at the same time, raise the frequency of failures. For example, when a state's regulators inspect each insurer's financial situation more aggressively and more frequently, more insurers will be identified as insolvent, but the extent of deficits left by insolvent insurers will be lower due to the earlier detection of insolvency. In this case, solvency regulations will benefit consumers when they do not cause a rise in premiums.

Summary

Table 2 summarizes welfare effects under fair premiums. Three different scenarios are discussed, i.e. the indemnity (or the probability of solvency) is either increased, not changed, or decreased.

TABLE 2
Welfare Effects Under Fair Premiums When the Indemnity (θ) or the Probability of Solvency (s) is Changed

	θ Increased	θ Not Changed	θ Decreased
s Increased	Benefit	Not Sure	Harm if Premiums Not Changed
s Not Changed	Benefit	No Effects	Harm
s Decreased	Benefit if Premiums Not Changed	Not Sure	Harm

Explanations of Results

The interpretation of Theorem 3 is that given the same premiums, consumers always prefer having a higher indemnity than having a higher probability of solvency. It is quite intuitive that consumers have such preferences. They purchase insurance so they can be indemnified in case they suffer losses. Thus, indemnity is the most important aspect of insurance to them.

To better understand why consumers have such preferences, particularly why the indemnification aspect is more important to them than the probability of solvency, let $V^* = V(q, s, \theta, \pi)$ be the consumer's indirect utility. Then, by the envelope theorem and using the fact that π is a function of both s and θ, one has:

$$\partial V^*/\partial\theta = \partial V/\partial\theta + \partial V/\partial\pi \frac{\partial\pi}{\partial\theta} \quad , \tag{16}$$

$$\partial V^*/\partial s = \partial V/\partial s + \partial V/\partial\pi \frac{\partial\pi}{\partial s} \quad . \tag{17}$$

In the above equations, the first term of the right side is the direct effect, which is always positive by Proposition 2. The whole second term is the price effect, which is negative. Consequently, the sign of $\partial V^*/\partial\theta$ or $\partial V^*/\partial s$ depends on which effect dominates. Under fair premiums, the direct effect of improving indemnification always dominates its price effect. However, the direct effect of improving solvency may be out-weighted by the price effect.

Regulatory Costs and Taxes

The results derived previously will be altered when consumers share regulatory costs through tax payments in addition to sharing the costs through extra premium payments. An insurer's contribution to the state's guaranty funds is tax-deductible. As a result, taxpayers pick up much of the cost of the state's guaranty funds.

In this case, guaranty funds and solvency regulation will benefit consumers only when regulatory costs are not too high.[17] Especially when a consumer has zero or even a negative benefit from insurance regulations, the consumer-taxpayer will be worse off under such regulations because of the need to share some regulatory costs.

One way to reduce the cost of regulation is for regulators to ask troubled insurers to cease operation sooner rather than later. It is in the consumers' best interest to prohibit an insurer from operating before the net worth of this insurer actually reaches a negative value. Any delay in closing financially troubled insurers will cause more burdens on guaranty funds and consequently will, in turn, raise the regulatory costs to consumers.[18]

Another way to reform the system of guaranty funds is to ask consumers to share some of the default risk of insurers. As indicated by Theorem 1, consumers are always better off when their insurance contracts are fully protected by guaranty funds. However, this would generally give rise to a moral hazard problem. When consumers are fully protected by such funds, they tend to be less sensitive to the issue of insurers' solvency. As an example, suppose there are two insurers, one is more solvent than the other and so charges a higher premium than the other insurer. In this case, practically all consumers will purchase their policies from the less solvent insurer realizing that their contracts do not have any default risk because of the full protection provided by the states' guaranty funds. As a result, the insurer with the higher solvency is driven out of the insurance market thereby, increasing the average insolvency of insurance firms. Consequently, the cost of the states' guaranty funds is raised and consumers are hurt because they have to share most of the costs of these funds. In contrast, when consumers' policies are not fully guaranteed, they are more careful in the selection of insurers. That, in turn, forces insurers to improve their probability of solvency and thus, reduce the cost of insolvencies. As a result, consumers bear lower regulatory costs and would be better off. Deductibles and co-payments have been widely used in insurance policies to reduce costs associated with consumers' moral hazard problems. The same principle can be applied to alleviate the moral hazard problem mentioned earlier.[19]

Conclusion

This paper shows that guaranty funds are always desirable, but solvency regulations may not be. The most certain value of solvency regulations, then, is to protect guaranty funds or to alleviate the disincentives they create as consumers will likely be less motivated to monitor insurers' behavior when protected by guaranty funds.

Federal intervention in insurance regulation has been called for to improve the solvency of insurance firms. This study indicates that such an effort might be unnecessary or even harmful because improving solvency may not be very beneficial. In particular, when such federal intervention causes higher regulatory costs, consumers are hurt. On the other hand, forming an interstate compact of guaranty funds or a national guaranty fund is attractive provided the pooling of states' funds would strengthen each state's capability to pay consumers when insurers are insolvent. However, such an effort may be harmful too when it leads to higher regulatory costs.

APPENDIX

This appendix gives and proves sufficient conditions that the indirect utility is an increasing function of the probability of solvency. First, a lemma is given.

Lemma 3.

$$\partial V^*/\partial s = q[U(W_2) - U(W_1)] - y^* q(1+r)(1-\theta)\frac{1}{\pi}[qsU'(W_2) + q(1-s)\theta U'(W_1)] \quad .$$

Proof: By the envelope theorem and using the definition of π, one has:

$$\frac{\partial V^*}{\partial s} = \partial V/\partial s + \partial V/\partial \pi \frac{\partial \pi}{\partial s}$$

$$= q[U(W_2) - U(W_1)] - qsU'(W_2)y^* q(1+r)(1-\theta) - q(1-s)U'(W_1)y^*(1+r)q \, (1-\theta) - (1-q)U'(W_0)y^* q(1+r)(1-\theta) \quad , \tag{1}$$

$$= q[U(W2) - U(W_1)] - y^* q(1+r)(1-\theta)[qsU'(W_2) + q(1-s)U'(W_1) + (1-q)U'(W_0)]$$

$$= q[U(W_2) - U(W_1)] - y^* q(1+r)(1-\theta)\frac{1}{\pi}[qsU'(W_2) + q(1-s)\theta U'(W_1)] \quad . \tag{2}$$

The last equation is by Lemma 1.

Theorem 4. $\partial V^*/\partial s \geq 0$ as s goes to 1 or θ goes to 0.
Proof: One only needs to prove the proposition at $s = 1$ or $\theta = 0$ due to the continuity of V^* with respective to s and θ. By Lemma 3, one has:

$$\begin{aligned} \partial V^*/\partial s &= q[U(W_2) - U(W_1)] - y^* q(1+r)(1-\theta)\frac{1}{\pi}[qsU'(W_2) + q(1-s)\theta U'(W_1)] \\ &= q[U(W_2) - U(W_1)] - y^* q(1-\theta)U'(W_2) \quad , \end{aligned} \tag{3}$$

Equation (3) is obtained by substituting $s = 1$ and $\pi = q(1+r)$.

Since $W_2 = W_1 + y^*(1-\theta)$, by the middle-value theorem, one has:

$$U(W_2) - U(W_1) = U'(W^*)(W_2 - W_1) = U'(W^*)y^*(1-\theta) \quad , \tag{4}$$

where $W_1 \leq W^* \leq W_2$. Because of $U'' < 0$, one has:

$$U'(W^*) \geq U'(W_2) \quad .$$

Thus, from (3) and (4), one has:

$$\partial V^*/\partial s = q[U'(W^*) - U'(W_2)]y^*(1-\theta) \geq 0 \quad .$$

Similarly, at $\theta = 0$, by Lemma 3, one has:

$$\partial V^*/\partial s = q[U(W_2) - U(W_1)] - y^* qU'(W_2) \quad . \tag{5}$$

Equation (5) uses $\pi = q(1+r)s$ for $\theta = 0$. Again, using $W_2 = W_1 + y^*$ at $\theta = 0$ and the middle-value theorem, one has:

$$U(W_2) - U(W_1) = U'(W^{**})(W_2 - W_1) = U'(W^{**})y^* \quad , \tag{6}$$

where $W_1 \leq W^{**} \leq W_2$. So, from (16) and (17), one has:

$$\partial V^*/\partial s = qy^*[U'(W^{**}) - U'(W_2)] \geq 0 \text{ for } U'(W^{**}) \geq U'(W_2) \quad .$$

Theorem 5. Under mean-variance preferences, $\partial V^*/\partial s > 0$ for $s > .5$.
Proof. By Lemma 3, one has:

$$\partial V^*/\partial s = q[U(W_2) - U(W_1)] - y^* q(1+r)(1-\theta)\frac{1}{\pi}[qsU'(W_2) + q(1-s)\theta U'(W_1)] \quad .$$

However, under mean-variance preferences, the consumer's optimal problem can be expressed as minimizing the variance of his wealth. Thus, $U^{(n)}(W) = 0$ for all $n > 2$. Then, by the Taylor's series expansion one has:

$$U(W_2) = U(W_1) + y^*(1-\theta)U'(W_1) + \frac{1}{2}y^{*2}(1-\theta)^2 U''(W_1) \quad , \tag{7}$$

and

$$U'(W_2) = U'(W_1) + y^*(1-\theta)U''(W_1) \quad . \tag{8}$$

By substituting (7) and (8) into the equation of $\partial V^*/\partial s$, one has:

$$\begin{aligned}\partial V^*/\partial s &= q[y^*(1-\theta)U'(W_1)+\frac{1}{2}y^{*2}(1-\theta)^2U''(W_1)]-y^*q(1+r)(1-\theta)\frac{1}{\pi}\\ &\quad [qs\{U'(W_1)+y^*(1-\theta)U''(W_1)\}+q(1-s)\theta U'(W_1)] \quad ,\end{aligned} \tag{9}$$

$$= qy^*(1-\theta)U'(W_1)[1-\frac{1}{\pi}q(1+r)(s+(1-s)\theta)]+\frac{1}{2}qy^{*2}(1-\theta)^2U''(W_1)[1-2qs(1+r)\frac{1}{\pi}] \quad , \tag{10}$$

$$= \frac{1}{2}qy^{*2}(1-\theta)^2U''(W_1)[1-2qs(1+r)\frac{1}{\pi}] \quad . \tag{11}$$

Equation (10) is because of $\pi = q(1+r)(s+(1-s)\theta)$. So, $\partial V^*/\partial s > 0$ at $\pi < 2qs(1+r)$. Or equivalently, $\partial V^*/\partial s > 0$ for $s > \frac{\theta}{1+\theta}$. Since $\theta \le 1$ implies $\frac{\theta}{1+\theta} \le .5$, one has:

$$\partial V^*/\partial s > 0 \text{ at } s > .5 \quad .$$

The above two theorems indicate that under one of three conditions increase in solvency will definitely benefit consumers. Such conditions include (1) the probability of solvency goes to 1, (2) the default indemnity is zero, and (3) the consumer has a mean-variance preference. However, the first two conditions are not realistic because each insurer has a positive probability of being insolvent no matter how it is regulated and because an insolvent insurer will always have some assets left when it is insolvent. In addition, the assumption that a consumer has a mean-variance preference is also too strong. Therefore, one would expect that solvency regulation hurts some consumers when it is used to prevent insolvency.

Footnotes

[1]In property-liability insurance, during 1969 and 1983, the average number of insolvencies was 9.8 per year, and the frequency of insolvency was only 0.3 percent. However, during 1984 and 1990, these numbers were 30 and 1.22 percent, respectively. During this same period, 480 property and liability insurers became insolvent [Best's Review, Property/Casualty Edition, March 1999, pp. 55-67].

[2]Forms of solvency regulations include minimum capital, surplus and reserve requirements; rate regulation; restrictions on investment; and financial inspection. Some states do not have rate regulation, but all states use the other forms of solvency regulation. Some consider guaranty funds as a form of solvency regulation, such as Munch and Smallwood [1980]. This paper, however, treats guaranty funds separately from solvency regulation. In order to distinguish the differences between their respective designed goals—one to raise indemnities and the other to prevent insolvency.

[3]Pauly, et. al. use data of 97 firms in the year of 1981; Frech and Samprone use states' aggregate data in the year of 1973.

[4]Here, the insurer's probability of insolvency is assumed to be unrelated to the insureds' loss. The model is not changed by relaxing such an assumption. To see this, let the probability of solvency s be s_1 when loss L occurs and be s_2 for no loss, $s_1 > s_2$. Then, s_2 does not appear on the expected utility. The only effect of the correlation between the individual risk and the insolvency of the insurer is to change the value of the probability of solvency. When the individual's loss adversely affects the insurer's solvency, the probability of solvency of the insurer will be lower than otherwise. Doherty and Schlesinger [1990] have shown that in this case, consumers with CARA (constant absolute risk aversion) will purchase less insurance coverage.

[5]When $\theta = 0$, $\pi = qs$, which is the result obtained by Doherty and Schlesinger [1990].

[6]When $\theta = 0$, $y^* < L$ for all consumers, which is the outcome that Doherty and Schlesinger have. In addition, when y^* is subjected to be $y^* \le L$, one will have $y^* = L$ for $q \le \frac{\theta}{s+(1-S)\theta}$. Posititive coverage means not a corner solution (i.e. $y^* = 0$, or $y^* > 0$).

[7]It can be shown that the sufficient condition for $(\partial y^*/\partial q) < 0$ globally holds if $U''' < 0$.

[8]Let $q^* = \frac{\theta}{s+(1-s)\theta'}$, then when θ becomes smaller, q^* becomes smaller and when s is smaller, q^* is larger. However, smaller q^* means that more consumers will be partially insured, and a larger q^* means that more consumers will be overly insured.

[9]Let $\pi = q(1+r)(s+(1-s)\theta)$ and π'$= q(1+r)(s+(1-s)\theta$'$)$, where θ'$-\theta = x$, be the premiums without and with guaranty funds, respectively. Then, the extra premium paid will be π'$-\pi = q(1+r)(1-s)x$.

[10]From 1984-89, states' guaranty funds assessments were on average $542 millions(in 1989's dollars) per year. Most of the money was paid to policyholders, while only a small proportion of the amount collected was used to lower regulatory costs [Harrington, 1991; A. M. Best's Insolvency Study, 1991].

[11]Munch and Smallwood [1980] find that minimum capital requirements significantly reduce the frequency of insolvency. Lee [1994] also finds that minimum capital and surplus requirements have significant deterrent effects on insolvencies. He also finds that restrictions on investment significantly reduce the number of failures.

[12]Under fair premiums, $\Delta\pi = q(1+r)((1-s)\ \Delta\theta + (1-\theta)\Delta s)$.

[13]Ross, et. al. [1993] give a theoretical analysis to show why a near bankrupt firm wants to adopt more risky strategies. Empirically, it is commonly known that bankrupt firms were involved in more risky strategies just before their bankruptcies.

[14]Insurers are required to file annual financial statements. In addition, most states examine insurers about once every three years, and the insurance commissioner has the right by law to examine insurers whenever deemed necessary.

[15]It can be shown that $(\partial V*/\partial\theta)|\pi = \text{const.} > 0$ as far as $\Delta\theta > [(1-\theta)/(1-s)](-\Delta s)$.

[16]The Federal Deposit Insurance (FDI) is guaranteed by the U.S. government and has little risk of being insolvent. However, payments from the state's guaranty funds are subject to the availability of funds. Assessments in most states are limited to 2 percent of the total written premiums.

[17]Denote Wg to be the regulatory costs including extra taxes a consumer shares, then the consumer's welfare under regulation will be $V(q, s', \theta', \pi', W - W_g)$, where s', θ', and π' are the relevant values under regulation. On the other hand, the consumer's welfare is $V(q, s, \theta, \pi, W)$ without regulation. The equivalent wealth gain W_e can be found by solving $V(q, s, \theta, \pi, W) = V(q, s', \theta', \pi', W - W_e)$. Therefore, it is obvious that the consumer will be better off under regulation only when $W_e > W_g$.

[18]State regulators tend to be reluctant to take quick actions against financially troubled insurers. Such reluctance could be due to political considerations or due to the lack of available information necessary to decide whether these insurers have any chance to recover. Such delay in closing financially troubled insurers has caused extra burdens on states' guaranty funds. However, wrongly identifying an insurer as insolvent will also cause problems. It will cause the insurer unnecessary loss of business because consumers will then be less confident in the insurer.

[19]Most states currently have an upper limit on how much can be paid from the guaranty fund per issued loss. In life-health insurance, the limitation is mostly around $100,000 per claim.

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