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Why Some Disaster Insurance Does not Exist

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Abstract

The failure of some disaster insurance market has been a very serious problem. This paper focuses on why disaster reinsurance fails and how that will affect the availability of primary disaster insurance. The insurer's unexpected costs are added to the expected costs associated with the insured event to illustrate the necessary and sufficient conditions for the existence of disaster insurance and reinsurance. Particularly, investors' negative response to an insurer's huge, disaster-related liability exposures may lead to availability problem unless the insurer's asset value losses in the financial market can be minimized. A large insurer may be more likely to withdraw from underwriting disaster insurance. Three different pricing schemes for disaster reinsurance contracts are investigated. The one which is based on the Option Pricing Theory is rejected because it leads to market failure.

KEYWORDS: disaster insurance, unexpected cost, options, asset value losses, fair price

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1. Introduction

Availability of disaster insurance and reinsurance has been a serious problem in recent years (Cummins et al., 2002; Froot, 1997, 1999; Kunreuther, 2006). For instance, following the big earthquake in 1989 in the Bay area in San Francisco, California, many residents in that area found they could no longer buy coverage of earthquake insurance from their original insurers. Following Hurricane Andrew, many residents on the eastern coast of the United States were declined disaster insurance coverage by most insurance companies. After the huge Katrina disaster in New Orleans, many insurers cancelled their offering of the relevant insurance coverage to the people in that area.

Several previous studies indicate that the unavailability of reinsurance is one of the major factors causing the primary insurers' unwillingness to continue their business. For instance, Berger, Cummins and Tennyson (1992) show that the unavailability of reinsurance coverage contributed to the crisis of liability insurance. Hershbarger (1994) demonstrates that inaccessibility of disaster reinsurance leads to the unavailability of disaster insurance.

Conventional explanations for the unavailability of insurance or reinsurance include non-diversification of risk, adverse selection, and moral hazard (Borch, 1990; Bum and Schlesinger, 2005). Given the nature of disaster insurance, these three factors may account, to some extent, for the non-existence of insurance or reinsurance, but they cannot fully explain the whole picture. The huge unexpected costs associated with disaster insurance significantly contributed to the failure of the disaster insurance market.

Such huge unexpected costs could lead to the insurers' insolvency. As Cummins (2007) stated that:

> "The magnitude of losses from Andrew (a Hurricane in 1992) in particular took insurers by surprise, and they drastically underestimated the financial impact of the hurricane even after the event took place. There were thirteen insurance company failures in 1992 and 1993 primarily attributable to Hurricane Andrew and three additional failures in Hawaii due to Hurricane Iniki, which also made landfall in 1992."

The unexpected costs include the insurer's extra market value loss associated with the extra decrease of its security (stocks and/or bonds) prices (Lamb, 1995, 1996)[1], decrease of demand for its business (some consumers may

[1] Even if the insurer does not have its stocks/bonds publicly traded, the firm may still bear extra unexpected financial loss as the interest rate the bank charges to the insurer will increase given its higher financial risk. More generally, the required rates of return of all types of investors (debt-

worry about the insurer's solvency so they refuse to buy coverage from the insurer) (Brown and Hoyt, 2000), unexpected and extra reimbursement to insureds (Cummins, 2007), and other factors such as extra payment to its employees working overtime and possible lawsuits. The insurer's security price will probably decrease dramatically if investors suddenly realize the firm's higher risking. One useful indicator of the firm's higher risk is the lowering of its financial rating by the professional financial rating firms.

A study by Cummins (2007) shows that there are unexpected and surprising losses when an insured disaster happens and these may still be underestimated even after they happen. Then insurers will suffer unexpected financial value losses after the insured disaster occurs, given the capital market efficiency hypothesis.

Unexpected costs affect the decision of insurers as to whether or not to be engaged in market competition. Aware of potential unexpected costs but unable to estimate and price them in the insurance contracts, some insurance firms may not want to do business in the area of disaster insurance. This may well explain why many primary insurers and reinsurers withdrew from the disaster insurance market.

Regarding the pricing of reinsurance, prior studies assume that the reinsurance market is perfectly competitive. In this paper, based on different conditions of insurance markets, three different pricing schemes for reinsurance are introduced. In addition, the necessary conditions for the existence of the reinsurance market and, therefore, the existence of the primary insurance market are given. In particular, this research shows that both primary and reinsurance markets may fail under a price derived from the Option Pricing Theory while these markets could exist otherwise.

Pricing of reinsurance has been widely studied. Borch (1974) uses the Equilibrium Theory to derive fair pricing for a reinsurance contract. Doherty and Garvens (1986) give the formula of reinsurance premiums by applying the Option Pricing Theory. Venter (1993) values the reinsurance contract by using the Arbitrage Pricing Theory. In this paper, a reinsurance contract is considered as a put option to the primary insurer with the price of the reinsurance contract being subsequently determined under three different conditions of the reinsurance market – a market monopolized either by the primary insurer or by the reinsurer and a market where competitive conditions prevail. Consequently, three different pricing schemes for the reinsurance contract are given – the minimum one, the maximum one, and the fair one.

The approach followed in this paper in addressing the problem of pricing of reinsurance differs from all of those suggested by previous studies, including

holders and stock-holders) will be higher when the insurer has a higher risk, so the cost of capital to the insurer will be higher.

that of Doherty and Garvens (1986) and Chen et al. (2001). The unexpected losses as well as expected losses to insurance firms resulting from the occurrence of the insured event are considered in price determination in this study. The expected losses include the expected reimbursement to the insureds and regular administrative expenses of the insurer.

This paper is organized as follows. Section 2 explores different pricing schemes of the reinsurance contract and gives conditions for the existence of reinsurance. Section 3 gives numerical examples, which show how both primary insurers and reinsurers may benefit by cooperating with each other. Section 4 demonstrates the effects of firm size, unexpected costs and the reinsurance premiums on the availability of insurance. The last section provides some concluding remarks.

2. Options and the Value of Reinsurance

2.1. The Choice of Strategies and the Value of Reinsurance

Suppose there are two strategies for the primary insurer. Under strategy 1 the insurer's net asset takes a random value X_1 with a distribution F_1, whereas under strategy 2, the firm's net asset has a random value X_2 with the distribution F_2, i.e. $X_1 \sim F_1(\alpha_1, \sigma_1)$ and $X_2 \sim F_2(\alpha_2, \sigma_2)$, where α_1 and σ_1 are the mean and standard deviation of X_1, respectively, and α_2 and σ_2 are the mean and standard deviation of X_2. In addition, it assumes that strategy 2 is riskier than strategy 1 with $\alpha_1 < \alpha_2$, and $\sigma_1 < \sigma_2$. Min $(X_2) < 0 <$ Min (X_1). In other words, the primary insurer will have a higher value on average under strategy 2, but its variance will also be higher. Moreover, the insurer may face the risk of bankruptcy under strategy 2.

Assume that the primary insurer will not choose strategy 2 without reinsurance either because it does not want to bear the risk[2] associated with this strategy or because regulation prohibits the insurer from doing so[3].

Now, suppose that a reinsurance firm offers an option to the primary insurer. Let Ex be the exercise price and O_p be the price of the option, Ex and $O_p \geq 0$. Then, one can illustrate what the price of the option is and under what conditions both the primary insurer and the reinsurer are willing to share risks and benefits.

The expected net benefit for the primary insurer from taking strategy 2 instead of 1 is $\Delta = Pro\ (X_2 \geq Ex) * E\ (X_2 \mid X_2 \geq Ex) + Ex * Pro\ (X_2 < Ex) - E\ (X_1)$

[2] An insurer will be able to bear such a disaster insurance risk when it can raise extra money from the financial market. Here, the assumption is that raising needed money is too costly for the insurer in the short time.

[3] Each insurer is required to have a minimum surplus ratio by the state's solvency regulation.

$-O_p$. By contrast, the expected payment which the reinsurer encounters is $P_m = E\,[(Ex - X_2); X_2 < Ex] = Ex * Pro\,(X_2 < Ex) - Pro\,(X_2 < Ex) * E\,(X_2 \mid X_2 < Ex)$.

Then, the necessary condition for the existence of reinsurance is $O_p \geq P_m$ and $\Delta \geq 0$. Consequently, one has:

(1) $Pro\,(X_2 \geq Ex) * E\,(X_2 \mid X_2 \geq Ex) + Ex * Pro\,(X_2 < Ex) - E\,(X_1) - O_p \geq 0$;

(2) $O_p \geq Ex * Pro\,(X_2 < Ex) - Pro\,(X_2 < Ex) * E\,(X_2 \mid X_2 < Ex)$.

Denote $\Delta_1 = E\,(X_2) - E\,(X_1)$ and

$\Delta_2 = O_p - Ex * Pro\,(X_2 < Ex) + Pro\,(X_2 < Ex) * E\,(X_2 \mid X_2 < Ex)$; thus,

$\Delta = \Delta_1 - \Delta_2$. From (1) and (2), one has $\Delta_1 = E\,(X_2) - E\,(X_1) \geq 0$.

In a reinsurance market monopolized by the primary insurer, the reinsurer earns zero profit, i.e. $\Delta_2 = 0$ or $O_p = Ex * Pro\,(X_2 < Ex) - Pro\,(X_2 < Ex) * E\,(X_2 \mid X_2 < Ex)$. By contrast, being a monopoly in the market, the reinsurer can seize all profits as much as it can, so $\Delta = 0$, i.e. $O_p = Ex * Pro\,(X_2 < Ex) + Pro\,(X_2 \geq Ex) * E\,(X_2 \mid X_2 \geq Ex) - E\,(X_1)$. In other words, $O_p = Ex * Pro\,(X_2 < Ex) - Pro\,(X_2 < Ex) * E\,(X_2 \mid X_2 < Ex)$. Denoted by $P_{\min}$ is a minimum premium that the reinsurer requires, and $O_p = Pro\,(X_2 \geq Ex) * E\,(X_2 \mid X_2 \geq Ex) + Ex * P\,(X_2 < Ex) - E\,(X_1)$; denoted by $P_{\max}$ is the maximum premium that the primary insurer is willing to pay for reinsurance. Let O'_p be the actual price of reinsurance, then the necessary conditions for the existence of reinsurance are $P_{\max} \geq O'_p \geq P_{\min}$.

In addition, one can use the Option Pricing Theory to derive a fair price for the option, where the exercise price[4] is Ex and the present value of the asset is $E\,(X_2)$. Let P_{fair} be the fair pricing from OPT[4]; one has $P_{\text{fair}} = E\,\{\text{Max}\,(Ex - X_2, 0)\}$, where E is the expectation operator. Thus, one has three different price schemes: $P_{\min}$, $P_{\max}$ and P_{fair}. The necessary condition for the existence of reinsurance and therefore insurance is that $P_{\min} \leq P_{\max}$. When the reinsurance is priced according to P_{fair}, the necessary conditions for the existence of both primary and reinsurance markets are $P_{\min} \leq P_{\text{fair}} \leq P_{\max}$.

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[4] There is no right price of insurance/reinsurance as people have argued. A fair insurance premium can be based on actuarial or financial pricing. The Capital Asset Pricing Model (CAPM), Option Pricing Theory (OPT) and Arbitrage Pricing Theory (ABT) have been widely used to derive fair insurance premiums. In this paper, we identify the reinsurance premium derived from the OPT as the fair reinsurance premium/price donated by P_{fair}.

Furthermore, one may consider the reinsurer's asset losses from the adverse event of the disaster. Let L_R be the asset value losses to the reinsurer when the reinsurer encounters the liability exposure, then the expected cost to the reinsurer is:

$$P_m = Ex * Pro\ (X_2 < Ex) - Pro\ (X_2 < Ex) * E\ (X_2 \mid X_2 < Ex) + L_R * Pro\ (X_2 < Ex)$$

$$= [Ex + L_R] * Pro\ (X_2 < Ex) - Pro\ (X_2 < Ex) * E\ (X_2 \mid X_2 < Ex)$$

Now, the necessary conditions for the existence of reinsurance become:

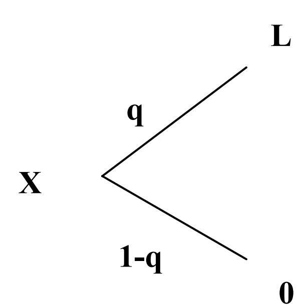
(1) $\Delta = Pro\ (X_2 \geq Ex) * E\ (X_2 \mid X_2 \geq Ex) + Ex * Pro\ (X_2 < Ex) - E\ (X_1) - O_p \geq 0$;

(2) $\Delta_2 = O_p - [Ex + L_R]\ Pro\ (X_2 < Ex) + Pro\ (X_2 < Ex) * E\ (X_2 \mid X_2 < Ex) \geq 0$.

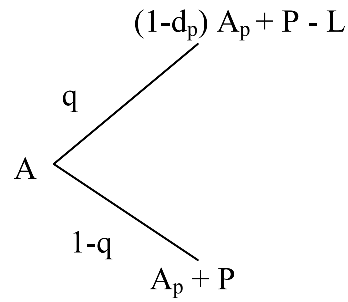
Satisfying condition (1) will motivate the primary insurer to underwrite policies while having condition (2) will make the reinsurer have an incentive to participate in the disaster insurance.

2.2. State Options and Pricing of Reinsurance

Suppose that the primary insurer has net assets A_p without underwriting disaster insurance. Assume further that disaster insurance has a loss distribution that has total claim indemnities of L with a probability of q.



Assume that the default rate of the primary insurer's asset is d_p when the insured event occurs and that the total premiums earned by the insurer are P. Then, the primary insurer's contingent assets have the following distribution after participating in the disaster insurance scheme:



The expected net assets to the primary insurer after underwriting the disaster insurance are $(1 - q\ d_p)\ A_p + (P - q\ L)$. The primary insurer may participate in issuing disaster insurance policies when

$$(1 - q\ d_p)\ A_p + (P - q\ L) - A_p > 0 \text{ or}$$

$$-q\ d_p\ A_p + (P - q\ L) > 0 \qquad (2.1)$$

From (2.1), one has $P > q\ L + q\ d_p\ A_p$. In other words, the premium that the insurer charges is at least as large as the expected payments, $q\ L$, plus the expected asset value losses, $q\ d_p\ A_p$.

The insurer's probability of insolvency assuming coverage for the disaster insurance is q which, if it is too high to satisfy certain regulatory requirements, will force the primary insurer to cede some premiums to satisfy solvency regulation.

Assume that the reinsurer has net assets A_R. The form of reinsurance takes excess of loss. Let Ex and O_p be, respectively, the exercise price and option price (called the premium of reinsurance)[5]. In other words, if $(1 - d_p)\ A_p + (P - L) - O_p < Ex$, the reinsurer will compensate the primary insurer. Also, assume that the default rate of the reinsurer's asset is d_R, when the reinsurer needs to pay claims to

[5] The relation between the stop loss and the put option is straight forward. Let $\overline{L}$ be the stop loss point, i.e. the reinsurer will compensate the primary insurer for all extra losses when the primary insurer's loss exceeds $\overline{L}$; then, $Ex = (1 - d_p)\ A_p + P - O_p - \overline{L}$.
Note that $d_R\ A_R$ occurs in the P_{fair} but it does not occur in the $P_{\max}$.

the primary insurer. Thus, the total costs to the reinsurer are $P_m = (Ex + d_R A_R) - E(X_2; X_2 < Ex) = q\ \{Ex + d_R A_R - [(1 - d_p) A_p + (P - L) - O_p]\}$. The reinsurer has an incentive to do business only when $O_p \geq P_m$.

Now, with reinsurance, the primary insurer's expected net assets are $(1 - q)\{(1 - d_p) A_p + (P - L) - O_p\} + q\ Ex$. Thus, the net benefit for the insurer by participating in the disaster insurance business is:

$$(1 - q)\ \{(1 - d_p) A_p + (P - L) - O_p\} + q\ EX - A_p \qquad (2.2).$$

Thus, the necessary conditions for the existence of disaster insurance are:

(1) $\Delta = (1 - q)(A_p + P - O_p) + q\ EX - A_p \geq 0$;

(2) $\Delta_2 = O_p - q\{Ex + d_R A_R - (1 - d_p) A_p - (P - L) + O_p\} \geq 0$.

When the primary insurer has the monopoly power over the reinsurer, one has $\Delta_2 = 0$. Hence,

$$P_{\min} = O_p = [q/(1 - q)][Ex + d_R A_R - (1 - d_p) A_p - P + L]. \qquad (2.3).$$

By contrast, when the reinsurance market is monopolized by the reinsurer, one has $\Delta = 0$, i.e.

$$P_{\max} = O_p = P - [q/(1 - q)]\ A_p + [q/(1 - q)]\ Ex \qquad (2.4).$$

Moreover, one can derive the price of the reinsurance using the Option Pricing Theory (OPT). Once again, let Ex be the exercise price of the option and let p be the hedging probability. Note that A_p is the present value of the primary insurer without option. Then, one has:

$$p\ [(1 - d_p)A_p + P - L] + (1 - p)(A_p + P) = A_p \qquad (2.5).$$

One can solve for p from the above equation and has:

$$p = P/(L + d_p A_p) \qquad (2.6).$$

Then, the expected value of the option is $p\ [Ex - (1 - d_p) A_p - P + L]$. In other words, the fair price of the option is:

$$P_{\text{fair}} = O_p = [P/(L + d_p A_p)]\ [Ex - (1 - d_p) A_p - P + L] + q\ d_R A_R \qquad (2.7).$$

In (2.7), the second term $q\ d_R\ A_R$ is the unexpected cost to the reinsurer. One may consider it as the transaction cost to the firm. Thus, one may view the first term in (2.7) as the fair price without transaction costs.

Now, one has three different price schemes for the reinsurance contract: the minimum price $P_{\min}$, the maximum one $P_{\max}$, and the fair one P_{fair}. The necessary condition for the existence of the disaster reinsurance and insurance, as stated before, is that $P_{\min} \leq P_{\max}$. In addition, when the reinsurance contract is priced with P_{fair}, the necessary conditions will be $P_{\min} \leq P_{\text{fair}} \leq P_{\max}$.

3. Numerical Examples

Assume that the primary insurer has a net asset value of $100 million. If it underwrites policies in a line like disaster insurance, it will, say, encounter potential direct losses of $300 million with a probability of 0.20. In addition, when the insured disaster occurs, assume that the insurer's assets are defaulted in the financial market and the default rate is 10%. To avoid its bankruptcy, without reinsurance the primary insurer must charge premiums totaling at least $210 million. Note that the expected claim costs are only $60 million (0.20 * $300 million) and the unexpected costs associated with the disaster are $2 million (0.20 * 10% * $100 million). Therefore, the insurance premiums could be as low as $62 million. Suppose that consumers are willing to be insured when the premiums they pay do not exceed $80 million. However, the primary insurer cannot take advantage of this business due to solvency regulation which considers the insurer's probability of insolvency of 20% when engaged in disaster insurance to be too high.

Suppose that a reinsurance firm has net assets of $300 million. The default rate of the reinsurer's assets is 5% when the reinsurer is engaged in disaster reinsurance. Let *Ex* be the exercise price the reinsurer offers to the primary insurer and O_p be the price of the option or the premium of reinsurance. From the discussion in Section 2.2, one knows that the minimum premium of reinsurance is:

$$P_{\min} = 0.2/.8\ [Ex + 5\% * 300 - 0.9 * 100 - P + 300] = 0.25\ [Ex - P] + 56.25$$

By contrast, the maximum premium is:

$$P_{\max} = P - 0.25 * 100 + 0.25\ Ex = P + 0.25\ Ex - 25$$

And, the fair premium derived from the OPT is:

$$P_{\text{fair}} = [P / (L + d_p\ A_p)]\ [Ex - (1 - d_p)\ A_p - (P - L)] + q\ d_R\ A_R$$

$= P / 310 [Ex - 90 + 300 - P] + 15 q = 3 + P / 310 (Ex + 210 - P)$.

To have $P_{max} \geq P_{min}$ one needs $P + 0.25 Ex - 25 \geq 0.25 [Ex - P] + 56.25$; or $P \geq 65$. In other words, the premiums the policyholders pay must be at least \$65 million dollars, which are the total costs, including the expected direct losses of \$60 million, the asset loss for the primary insurer of \$2 million, and the asset loss for the reinsurer of \$3 million.

In case $Ex = 0$, one has:

$P_{min} = 56.25 - 0.25 P$; $P_{max} = P - 25$; and $P_{fair} = 3 + P (210 - P) / 310$.

When $Ex = 100$, one has:

$P_{min} = 81.25 - 0.25 P$; $P_{max} = P$; and $P_{fair} = 3 + P (310 - P) / 310$.

Furthermore, one can explore the pricing of reinsurance given the insurance premiums paid by the policyholders. Suppose that $P = 65$, then one has:

$P_{min} = P_{max} = 0.25 Ex + 40$, and

$P_{fair} = 3 + 65 / 310 (Ex + 145) = 33.40 + 0.21 Ex$.

Thus, there may be a situation where $P_{fair} < P_{min}$. In other words, if the reinsurance premium is set to equal the P_{fair} determined from the OPT, the reinsurer may suffer a net loss and, therefore, it will have no incentive to do business.

In fact, it can be verified that P must equal 79.28 to guarantee that $P_{fair} \geq P_{min}$ for $Ex = 0$. If $P = 79.28$ and $Ex = 0$, then $P_{min} = P_{fair} = 36.43$, and $P_{max} = 54.28$.

It is interesting to note that a fair reinsurance premium derived using the OPT leads to the insurance market failure. To better understand why this happens, one can derive the necessary and sufficient condition for having $P_{fair} \geq P_{min}$. By substituting (2.3) and (2.7) into inequality $P_{fair} \geq P_{min}$, one can easily have:

$$[P / (L + d_p A_p) - q / (1 - q)] [Ex - (1 - d_p) A_p - (P - L)] \geq q / (1 - q) (q d_R A_R) \quad (2.8)$$

Condition (2.8) is the necessary and sufficient condition for the coexistence of both reinsurance and insurance markets.

As $(1 - d_p) A_p + (P - L)$ will be the net assets of the primary insurer when insured loss occurs and Ex is the exercise price of the reinsurance option, $Ex - (1 - d_p) A_p - (P - L)$ should be positive in order for the primary insurer to make its claim to the reinsurer[6]. Therefore, to have (2.8), one must have $[P / (L + d_p A_p) - q / (1 - q)] \geq 0$ or have:

$$P / (L + d_p A_p) \geq q / (1 - q) \qquad (2.9).$$

The above inequality is a necessary condition for $P_{\text{fair}} \geq P_{\text{min}}$, but not sufficient.

Remember that based on (2.1), the condition for a primary insurer to underwrite the insurance policy is:

$$P / (L + d_p A_p) \geq q \qquad (2.10)$$

It is obvious that (2.9) implies (2.10) but the opposite is not true. In other words, if the reinsurer is able to charge its reinsurance premium (P_{fair}) which will be enough to compensate its expected underwriting loss plus its possible asset default loss (P_{min}), i.e. $P_{\text{fair}} \geq P_{\text{min}}$, then the primary insurer will also be motivated to issue the policy to the insured. However, even if the primary insurer is willing to do business because of (2.10), the reinsurer will not be interested in such reinsurance business if (2.9) is not met.

Furthermore, satisfying (2.9) is not enough to motive the reinsurer. The right side of (2.8) indicates whether the reinsurer is willing to do this type of business depends on the odds of the disaster, $q / (1 - q)$, and the expected total asset default ($q\, d_R\, A_R$). When such odds are very high like 100%, and the expected total asset default is huge, one can see that no reinsurer could be willing to underwrite policies, or reinsurers would leave the disaster insurance market after they observe such a scenario.

4. Unexpected Costs, Pricing of Reinsurance and Availability of Insurance

The previous section indicates that the reinsurance market and thus the insurance market may fail because of the unexpected costs (asset value losses) and pricing of reinsurance.

First, unexpected costs may lead to the failure of the insurance markets because of the following reasons. If the insurance premiums the policyholders need to pay include the expected losses plus unexpected losses, when there are huge unexpected losses from the financial market associated with the insured events, the loading factor in the premiums charged to policyholders will be so big that no consumer would want to purchase insurance. By contrast, when insurance

[6] In fact, through this paper, it is assumed that $(1 - d_p) A_p + (P - L)$ will be negative. Thus, $Ex - (1 - d_p) A_p - (P - L)$ should be always positive as Ex is non-negative.

firms cannot, because of rate regulation, adjust their premiums to cover the unexpected costs, no firm will have the incentive to underwrite disaster insurance policies.

Secondly, the pricing of reinsurance using the OPT may cause the failure of the reinsurance market. Note that the OPT is based on the assumption that the reinsurance market is perfectly competitive or there is no arbitrage. If so, the price of the option is whatever its value will be to the primary insurer. As a result, the price derived from the OPT does not guarantee the existence of the reinsurance market because it ignores profit incentives of the reinsurers.

The example demonstrated in Section 3 shows that one may have $P_{\text{fair}} < P_{\text{min}}$; in other words, a reinsurer may not be compensated enough when the price is set up according to P_{fair}. By contrast, one can find a case with $P_{\text{fair}} > P_{\text{max}}$. This can happen when the reinsurer has huge unexpected costs denoted by $d_R A_R$.

In addition, the size of the firm affects the availability of insurance. To underwrite policies like the ones in disaster insurance, a firm must have enough assets. However, a big firm may not have any competitive advantages due to the unexpected costs. To see this, let two firms have assets A_1 and A_2, respectively, with $A_1 > A_2$. And let d_1 and d_2 be the default rate of their assets when the insured event occurs. Thus, the total defaulted assets to the big firm are $d_1 A_1$ and the total defaulted assets to the small firm are $d_2 A_2$. The big firm will have its advantage or lower unexpected costs only when $d_1 A_1 < d_2 A_2$, or $d_1 < d_2 A_2 / A_1$. Assume that the big firm has twice the assets of the small firm; then, the big firm will have the lower unexpected costs only when its default rate is less than half of that of the small firm.

5. Conclusions

By including insurers' unexpected costs as well as expected costs associated with the insured event, this paper illustrates conditions for the existence of disaster insurance and reinsurance. The study particularly demonstrates that the financial markets' negative response to an insurer's huge liability exposures related to the disaster could prohibit the insurer from continuing its coverage to the policyholders.

A larger insurer is not necessarily more willing to engage in underwriting disaster insurance unless it can minimize its value losses in the financial market caused by the liability exposures. In fact, there is evidence that larger insurers may more likely leave the market after the disaster[7].

[7] It was reported that larger insurers left from the catastrophic insurance market after hurricanes in Miami but new small insurers entered the market (Klein, 2008).

Moreover, this paper provides three different pricing schemes for reinsurance – a maximum price, a minimum price, and a fair price – based on certain market conditions. This paper demonstrates that a fair reinsurance premium derived from the OPT may lead to the failure of all insurance markets.

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